REFLECTIONS

ON THE

MOTIVE POWER OF HEAT

AND ON

MACHINES FITTED TO DEVELOP THAT POWER.

FROM THE ORIGINAL FRENCH OF

N.-L.-S. CARNOT,
GRADUATE OF THE POLYTECHNIC SCHOOL.

EDITED BY

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Director of Sibley College, Cornell University;
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etc., etc., etc.

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DEDICATED

TO

Sadi Carnot,

PRESIDENT OF THE FRENCH REPUBLIC,

THAT DISTINGUISHED MEMBER OF THE PROFESSION OF ENGINEERING
WHOSE WHOLE LIFE HAS BEEN AN HONOR TO HIS
PROFESSION AND TO HIS COUNTRY;

AND WHO, ELEVATED TO THE HIGHEST OFFICE WITHIN THE GIFT OF THE
FRENCH NATION,
HAS PROVEN BY THE QUIET DIGNITY AND THE EFFICIENCY WITH WHICH
HE HAS PERFORMED HIS AUGUST DUTIES THAT HE IS
A WORTHY MEMBER OF A NOBLE FAMILY,
ALREADY RENDERED FAMOUS BY AN EARLIER SADI CARNOT,
NOW IMMORTAL IN THE ANNALS OF SCIENCE,

AND IS HIMSELF DESERVING OF ENROLMENT IN A LIST OF GREAT MEN
WHICH INCLUDES THAT OTHER DISTINGUISHED ENGINEER,
OUR OWN FIRST PRESIDENT,

GEORGE WASHINGTON.

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PUBLISHERS' NOTE.

THE *raison d'être* of the following translation of the famous work of Carnot is not the usual one, either with the Publishers or the Editor—expectation of gain in either purse or fame. Neither could reasonably be anticipated from the reproduction of the work of an author of more than a half-century ago, in a field then unrecognized, and to-day familiar to but few; and especially when, as is in this case the fact, the work itself has been long out of date as a scientific authority, even had it ever held such a position. It could not be presumed that a very large proportion of even the men of science of the English-speaking world would be sufficiently familiar with the subject, or interested in its origin, to purchase such a relic of a primitive period as is this little book. Nor could the translation of the work, or the gathering together by the Editor of related matter, be supposed likely to be productive of any form of compensation. The book is published as matter of limited but most intense scientific interest, and on that score only.

It has seemed to the Editor and to the Publishers that the product of the wonderful genius of Carnot,—the great foundation-stone of one of the most marvellous and important of modern sciences, the first statement of the grand though simple laws of Thermodynamics,—as illustrated in this one little treatise, should be made accessible to all who desire to study the work in English, and preserved, so far as its publication in this form could accomplish it, as a permanent memorial, in a foreign tongue, of such grand truths, and of such a great genius as was their discoverer. It is with this purpose that Publishers and Editor have co-operated in this project.

The book consists, as will be seen on inspection, of the translation of Carnot's *Réflexions sur la Puissance Motrice du Feu,* preceded by a notice written by the Editor calling attention to its remarkable features, and its extraordinary character as the product of a most remarkable genius; and by a biographical sketch of the great author, written by his brother, Mons. Henri Carnot, which sketch we find in the French copy of the work as published by Gauthier-Villars, the latest reproduction of the book in the original tongue. To the main portion of the book, Carnot's *Réflexions,* is appended the celebrated paper of Sir

William Thomson, his "Account of Carnot's Theory," in which that great physicist first points out to the world the treasure so long concealed, unnoticed, among the scientific literature, already mainly antiquated, of the first quarter of the nineteenth century. The distinguished writer of this paper has kindly interested himself in the scheme of the Editor, and has consented to its insertion as a natural and desirable commentary upon the older work, and especially as exhibiting the relations of the fundamental principles discovered and enunciated by Carnot to the modern view of the nature of thermodynamic phenomena—relations evidently understood by that writer, but not by the leaders of scientific thought of his time, and therefore ignored by him in the construction of his new science.

The Appendix contains a number of Carnot's own notes, too long to be inserted in the body of the paper in its present form, and which have therefore been removed to their present location simply as a matter of convenience in book-making.

The dedication of the work to the grand-nephew of the author, who by a singular coincidence happens to-day to occupy the highest position that any citizen can aspire to reach in that

now prosperous Republic, will be recognized as in all respects appropriate by every reader of the work of the earlier Sadi Carnot who is familiar with the character, the history, the attainments, the achievements, of the later Sadi Carnot in so many and widely diverse fields. The Carnot talent and the Carnot character are equally observable in both men, widely as they are separated in time and in the nature of their professional labors. Both are great representatives of a noble family, whose honor and fame they have both splendidly upheld.

The Publishers offer this little book to its readers as a small, yet in one sense not unimportant, contribution to the great cause of modern science, as a relic, a memorial, a corner-stone.

NOTE BY THE EDITOR.

"*Je me suis proposé de grands desseins dans ce petit ouvrage*," as Bernardin de Saint-Pierre says in the preface to his pathetic story of *Paul et Virginie.* I have sought to present to the great English-speaking world the work of a genius hitherto only known to a few men of science, and not well known, even among the people of France, for whose credit he has done so much. In placing before the readers of this translation his book—small of size but great in matter as it is—I feel that I have accomplished an easy task, but one of real importance. I have been asked, as Corresponding Member for the United States of the Société des Ingénieurs Civils de France, to communicate to my colleagues scientific and professional memoirs and whatever may be of interest to them—"*en un mot, que nous resserions les liens qui font des ingénieurs en général une seule famille.*" That were a pleasant task; but a grander and a more agreeable one still is that of bringing "nearer in heart and thought" the members of that still larger community, the men of science of the world, and of weaving still

more firmly and closely those bonds of kindly thought and feeling which are growing continually more numerous and stronger as the nations are brought to see that humanity is larger and more important than political divisions, and that the labors of educated men and of the guiding minds in the great industries are constantly doing more to promote a true brotherhood of mankind than ever have, or ever can, the greatest statesmen.

When the wonderful intellectual accomplishments of men like the elder Sadi Carnot become known and appreciated by the world, much more will have been accomplished in this direction. It is perhaps from this point of view that the importance of such work will be most fully recognized. When the little treatise which is here for the first time published in English becomes familiar to those for whom it is intended, it will be, to many at least, a matter of surprise no less than pleasure to discover that France has produced a writer on this now familiar subject whose inspiration anticipated many of the principles that those founders of the modern science, Rankine and Clausius, worked out through the tedious and difficult methods of the higher mathematics, and which were hailed by their contemporaries as marvellous discoveries.

The time will come, if it has not already arrived, when every man of science throughout the world, no less than every patriotic son of France, will unite in doing honor to this Carnot, who has thus gained fame for himself, increased distinction for his family, and honor of the noblest sort for his country.

I.

THE WORK OF SADI CARNOT.

By the Editor.

Nicolas-Leonard-Sadi Carnot was, perhaps, the greatest genius, in the department of physical science at least, that this century has produced. By this I mean that he possessed in highest degree that combination of the imaginative faculty with intellectual acuteness, great logical power and capacity for learning, classifying and organizing in their proper relations, all the facts, phenomena, and laws of natural science which distinguishes the real genius from other men and even from the simply talented man. Only now and then, in the centuries, does such a genius come into view. Euclid was such in mathematics; Newton was such in mechanics; Bacon and Compte were such in logic and philosophy; Lavoisier and Davy were such in chemistry; and Fourier, Thomson, Max-

well, and Clausius were such in mathematical physics. Among engineers, we have the examples of Watt as inventor and philosopher, Rankine as his mathematical complement, developing the theory of that art of which Watt illustrated the practical side ; we have Hirn as engineer-experimentalist, and philosopher, as well ; Corliss as inventor and constructor ; and a dozen creators of the machinery of the textile manufactures, in which, in the adjustment of cam-work, the highest genius of the mechanic appears.

But Carnot exhibited that most marked characteristic of real genius, the power of applying such qualities as I have just enumerated to great purposes and with great result while still a youth. Genius is not dependent, as is talent, upon the ripening and the growth of years for its prescience ; it is ready at the earliest maturity, and sometimes earlier, to exhibit its marvellous works ; as, for example, note Hamilton the mathematician and Mill the logician ; the one becoming master of a dozen languages when hardly more than as many years of age, reading Newton's Principia at sixteen and conceiving that wonderful system, quaternions, at eighteen ; the other competent to begin the study of Greek at three, learning Latin at seven and reading Plato before he

was eight. Carnot had done his grandest work of the century in his province of thought, and had passed into the Unseen, at thirty-six; his one little volume, which has made him immortal, was written when he was but twenty-three or twenty-four. It is unnecessary, here, to enter into the particulars of his life; that has been given us in ample detail in the admirable sketch by his brother Henri, herewith published. It will be quite sufficient to indicate, in a few words, what were the conditions amid which he lived and the relation of his work to that great science of which it was the first exposition.

At the time of Carnot, the opinion of the scientific world was divided, as it had been for centuries, on the question of the true nature of heat and light, and as it still is, to a certain extent, regarding electricity. On the one hand it was held by the best-known physicists that heat is a substance which pervades all bodies in greater or less amount, and that heating and cooling are simply the absorption and the rejection of this "imponderable substance" by the body affected; while, on the other hand, it was asserted by a small but increasing number that heat is a "mode of motion," a form of energy, not only imponderable, but actually immaterial; a quality

of bodies, not a substance, and that it is identical, in its nature, with other forms of recognizable energy, as, for example, mechanical energy. A quarter of a century before Carnot wrote, the experiments of Rumford and of Davy had been crucial in the settlement of the question and in the proof of the correctness of the second of the two opposing parties ; but their work had not become so generally known or so fully accepted as to be acknowledged as representative of the right views of the subject. The prevalent opinion, following Newton, was favorable to the first hypothesis; and it was in deference to this opinion that Carnot based his work on an inaccurate hypothesis; though, fortunately, the fact did not seriously militate against its value or his credit and fame.

"With true philosophical caution, he avoids committing himself to this hypothesis ; though he makes it the foundation of his attempt to discover how work is produced from heat." *

The results of Carnot's reasoning are, fortunately, mainly independent of any hypothesis as to the nature of heat or the method or mechanism of development and transfer or transformation of its energy. Carnot was in error in assuming no

* Tait: Thermodynamics, p. 13.

loss of heat in a completed cycle and in thus ignoring the permanent transformation of a definite proportion into mechanical energy; but his proposition that efficiency increases with increase of temperature-range is still correct; as is his assertion of its independence of the nature of the working substance.

Carnot's "*Réflexions sur la Puissance Motrice du Feu*," published in 1824, escaped notice at the time, was only now and then slightly referred to later, until Clapeyron seized upon its salient ideas and illustrated them by the use of the Watt diagram of energy, and might, perhaps, have still remained unknown to the world except for the fact that Sir William Thomson, that greatest of modern mathematical physicists, fortunately, when still a youth and at the commencement of his own great work, discovered it, revealed its extraordinary merit, and, readjusting Carnot's principles in accordance with the modern views of heat-energy, gave it the place that it is so well entitled to in the list of the era-making books of the age. But it still remained inaccessible to all who could not find the original paper until, only a few years since, it was reprinted by Gauthier-Villars, the great publishing house of Paris, accompanied by a biographical sketch by the younger brother, which

it has been thought wise to reproduce with the translation of Carnot's book. In making the translation, also, this later text has been followed; and now, for the first time, so far as is known to the writer, the work of Carnot is made accessible to the reader in English.

The original manuscript of Carnot has been deposited by his brother in the archives of the French Academy of Sciences, and thus insured perpetual care. The work of Carnot includes not only the treatise which it is the principal object of this translation to give to our readers, but also a considerable amount of hitherto unpublished matter which has been printed by his brother, with the new edition of the book, as illustrative of the breadth and acuteness of the mind of the Founder of the Science of Thermodynamics.

These previously unpublished materials consist of memoranda relating to the specific heats of substances, their variations, and various other facts and data, and principles as well; some of which are now recognized as essential elements of the new science, even of its fundamental part. The book is particularly rich in what have been generally supposed to be the discoveries of later writers, and in enunciations of principles now recognized as those forming the base and the sup-

porting framework of that latest of the sciences. As stated by Tait, in his history of Thermodynamics, the "two grand things" which Carnot originated and introduced were his idea of a "cycle" and the notion of its "reversibility," when perfect. "Without this work of Carnot, the modern theory of energy, and especially that branch of it which is at present by far the most important in practice, the dynamical theory of heat, could not have attained its now enormous development." These conceptions, original with our author, have been, in the hands of his successors, Clausius and other Continental writers, particularly, most fruitful of interesting and important results; and Clapeyron's happy thought of so employing the Watt diagram of energy as to render them easy of comprehension has proved a valuable aid in this direction.

The exact experimental data needed for numerical computations in application of Carnot's principles were inaccessible at the date of his writing; they were supplied, later, by Mayer, by Colding, by Joule, and by later investigators. Even the idea of equivalence, according to Mon. Henri Carnot, was not originally familiar to the author of this remarkable work; but was gradually developed and defined as he progressed with his philosophy. It is sufficiently distinctly enunciated in his later

writings. He then showed a familiarity with those notions which have been ascribed generally to Mayer and which made the latter famous, and with those ideas which are now usually attributed to Joule with similar result. He seems actually to have planned the very kind of research which Joule finally carried out. All these advanced views must, of course, have been developed by Carnot before 1832, the date of his illness and death, and ten or fifteen years earlier than they were made public by those who have since been commonly considered their discoverers. These until lately unpublished notes of Carnot contain equally well-constructed arguments in favor of the now accepted theory of heat as energy. While submitting to the authority of the greatest physicists of his time, and so far as to make their view the basis of his work, to a certain extent, he nevertheless adhered privately to the true idea. His idea of the equivalence of heat and other forms of energy was as distinct and exact as was his notion of the nature of that phenomenon. He states it with perfect accuracy.

In making his measures of heat-energy, he assumes as a unit a measure not now common, but one which may be easily and conveniently reduced to the now general system of measurement. He

takes the amount of power, required to exert an energy equal to that needed to raise one cubic meter of water through a height of one meter, as his unit; this is 1000 kilogrammeters, taken as his unit of motive power; while he says that this is the equivalent of 2.7 of his units of heat; which latter quantity would be destroyed in its production of this amount of power, or rather work. His unit of heat is thus seen to be $1000 \div 2.7$, or 370 kilogrammeters. This is almost identical with the figure obtained by Mayer, more than ten years later, and from presumably the same approximate physical data, the best then available, in the absence of a Regnault to determine the exact values. Mayer obtained 365, a number which the later work of Regnault enabled us to prove to be 15 per cent. too low, a conclusion verified experimentally by the labors of Joule and his successors. Carnot was thus *a* discoverer of the equivalence of the units of heat and work, as well as the revealer of the principles which have come to be known by his name. Had he lived a little longer, there can be little doubt that he would have established the facts, as well as the principles, by convincing proof. His early death frustrated his designs, and deprived the

world of one of its noblest intellects, just when it was beginning its marvellous career.

The following sentence from Carnot illustrates in brief his wonderful prescience; one can hardly believe it possible that it should have been written in the first quarter of the nineteenth century: "*On peut donc poser en thèse générale que la puissance motrice est en quantité invariable dans la Nature; qu'elle n'est jamais, à proprement parler, ni produite, ni détruite. Á la vérité, elle change de forme, c'est a dire qu'elle produit tantôt un genre de mouvement, tantôt un autre; mais elle n'est jamais anéantie.*" It is this man who has probably inaugurated the development of the modern science of thermodynamics and the whole range of sciences dependent upon it, and who has thus made it possible to construct a science of the energetics of the universe, and to read the mysteries of every physical phenomenon of nature; it is this man who has done more than any contemporary in his field, and who thus displayed a more brilliant genius than any man of science of the nineteenth century: yet not even his name appears in the biographical dictionaries; and in the Encyclopædia Britannica it is only to be found incidentally in the article on Thermodynamics.

Throughout his little book, we find numerous

proofs of his clearness of view and of the wonderful powers of mind possessed by him. He opens his treatise by asserting that "*C'est à la chaleur que doivent être attribués les grands mouvements qui frappent nos regards sur la terre; c'est à elle que sont dues les agitations de l'atmosphère, l'ascension des nuages, la chut des pluies et des autres météores, les courants d'eau qui sillonnent la surface du globe et dont l'homme est parvenue à employer pour son usage une faible partie; enfin les tremblements de terre, les éruptions volcanique reconnaissent aussi pour cause le chaleur.*"

Carnot was the first to declare that the quantity of work done by heat, in any given case of application in the heat-energy, is determined solely by the range of temperature through which it fell in the operation, and is entirely independent of the nature of the working substance chosen as the medium of transfer of energy and the vehicle of the heat. His assumption of the materiality of heat led, logically, to the conclusion that the same quantity of heat was finally stored in the refrigerator as had, initially, left the furnace, and that the effect produced was a consequence of a fall of temperature analogous to a fall of water; but, aside from this error—which he himself was evi-

dently inclined to regard as such,—his process and argument are perfectly correct.*

Throughout his whole work are distributed condensed assertions of principles now well recognized and fully established, which indicate that he not only had anticipated later writers in their establishment, but that he fully understood their real importance in a theory of heat-energy and of heat-engines. In fact, he often italicizes them, placing them as independent paragraphs to more thoroughly impress the reader with their fundamental importance. Thus he says: "*Partout où il existe une différence de température, il peut y avoir production de puissance motrice;*" and again, this extraordinary anticipation of modern science: "*le maximum de puissance résultant de l'emploi de la vapeur est aussi le maximum de puissance motrice réalisable par quelque moyen que ce soit.*"

"*La puissance motrice de la chaleur est indépendante des agents mis en œuvre pour la réaliser; sa quantité est fixée uniquement par les tempera-*

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*Account of Carnot's Theory of the Motive Power of Heat; Sir Wm. Thomson; Trans. Roy. Soc. of Edinburgh, xvi. 1849; and Math. and Phys. Papers, xli. vol. 1 (Cambridge, 1882), p. 113. In this paper the corrections due to the introduction of the dynamic theory are first applied

tures des corps entre lesquels se fait, en dernier résultat, le transport du calorique."

"*Lorsqu'un gaz passe, sans changer de température, d'un volume et d'une pression déterminé a un autre pression également déterminées, le quantité de calorique absorbée ou abandonnée est toujours la même, quelle que soit la nature du gaz choisi comme sujet d'expérience.*"

Perhaps as remarkable a discovery as any one of the preceding (and one which, like those, has been rediscovered and confirmed by later physicists; one which was the subject of dispute between Clausius, who proved its truth by the later methods which are now the source of his fame, and the physicists of his earlier days, who had obtained inaccurate measures of the specific heats of the gases;—values which were finally corrected by Regnault, thus proving Carnot and Clausius to be right) is thus stated by Carnot, and is italicized in his manuscript and book:

"*La différence entre le chaleur spécifique sous pression constante et la chaleur spécifique sous volume constant est la même pour tous les gaz.*"

He bases his conclusion upon the simplest of thermodynamic considerations. He says that the increase of volumes with the same differences of temperature are the same, according to Gay-Lussac

and Dalton; and that, therefore, according to the laws of thermodynamics as he has demonstrated them, the heat absorbed with equal augmentations of volume being the same, the two specific heats are constant, and their difference as well. As will be seen on referring to the text, he bases upon this principle a determination of the specific heats of constant volume, taking as his values of the determined specific heats of constant pressure those of Delaroche and Bérard, making the constant difference 0.300, that of air at constant pressure being taken as the standard and as unity. The establishment of this point, in the face of the opposition, and apparently of the facts, of the best physicists of his time, was one of those circumstances which did so much to win for Clausius his great fame. How much greater credit, then, should be given Carnot, who not only anticipated the later physicists in this matter, but who must have enunciated his principle under far more serious discouragements and uncertainty!

It must be remembered, when reading Carnot, that all the "constants of nature" were, in his time, very inaccurately ascertained. It is only since the time of Regnault's grand work that it has been the rule that such determinations have been published only when very exactly determined. No

change has been attempted in Carnot's figures, in any respect; as it would be far less satisfactory to read a paraphrased work, and the exact figures are now easily accessible to every one, and his computations may all be made, if desired, on the basis of modern data. Sir William Thomson has already performed this task in the paper appended.

Throughout the whole of this treatise, small as it is, we find distributed a singular number of these anticipations of modern thermodynamic principles. Studying the relation of heat-energy to work done, he concludes:

"*La chute du calorique produit plus de puissance motrice dans les degrés inférieurs que dans les degrés supérieurs.*"

We to-day admit that, since the one degree at a low temperature, and the corresponding quantity of heat, are larger fractions of the total temperature, and the total heat stored in the substance, than the one degree at a higher point on the scale of absolute temperature, this principle of Carnot has become obvious.

In the enunciation of the essential principles of efficiency of the heat-engine, we find the proofs of this same wonderful prescience. He asserts that, for best effect: "(1) The temperature of the working fluid must be raised to the highest degree

possible, in order to secure a commensurate range of temperature; (2) The cooling must be carried to the lowest point on the scale that may be found practicable; (3) The passage of the fluid from the upper to the lower limit of temperature must be produced by expansion;" i.e., "it is necessary that the cooling of the gas shall occur spontaneously by its rarefaction; "which is simply his method of stating the now universally understood principle that, for highest efficiency, the expansion must be adiabatic, from a maximum to a minimum temperature. He goes on to explain these principles, and then says that the advantage of high-pressure engines lies "*essentiellement dans la faculté de rendre utile une plus grand chute de calorique.*" This principle, as a practical system of operation, had already, as he tells us, been enunciated by M. Clément, and had been practised, as we well know, since the days of its originator, Watt; but Carnot saw clearly the thermodynamic principle which underlies it, and as clearly states it, for the first time.

He sees clearly, too, the reasons for the attempts of Hornblower and of Woolf, premature as they proved and as he also sees, in the introduction of the compound engine, and even suggests that this idea might be still further developed by the use

of a triple-expansion engine, a type which is to-day just coming into use, more than a half-century after Carnot's date. He recognizes the advantages of the compound engine in better distribution of pressures and in distribution of the work of expansion, but does not, of course, perceive the then undiscovered limitation of the efficiency of the simple engine, due to "cylinder condensation," which has finally led, perhaps more than any other circumstance, to its displacement so largely by the multi-cylinder machine. No one has more exactly and plainly stated the respective advantages to be claimed for air and the gases, used as working fluids in heat-engines, than does Carnot; nor does any one to-day better recognize the difficulties which lie in the path to success in that direction, in the necessity of finding a means of handling them at high temperatures and of securing high mean pressures.

His closing paragraph shows his extraordinary foresight, and the precision with which that wonderful intellect detected the practical elements of the problem which the engineer, from the days of Savery, of Newcomen, and of Watt, has been called upon to study, and the importance of the work, which he began, in the development of a theory of the action, or of the operation, of the

heat-engines, which should give effective assistance in the development of their improved forms :

"On ne doit pas se flatter de mettre jamais à profit, dans la pratique, toute la puissance des combustibles. Les tentatives que l'on ferait pour approcher ce résultat seraient même plus nuisible qu'utile, si elles faisait négliger d'autres considérations importantes. L'économie du combustibles n'est qu'une des conditions à remplir par les machines à feu ; dans beaucoup de circonstances, elle n'est que secondaire : elle doit souvent céder le pas à la sûreté, à la solidite, à la durée de la machine, au peu de place qu'il faut lui faire occuper, au peu de frais de son établissement, etc. Savoir apprécier, dans chaque cas, à leur juste valeur, les considérations de convenance et d'économie qui peuvent presenter ; savoir discerner les plus importantes de celles qui sont seulement accessoires, les balancier toutes con venablement entre elles, afin de parvenir, par les moyens les plus faciles, au meilleur résultat : tel doit être le principal talent de l'homme appelé à diriger, à co-ordonner entre eux les travaux de ses semblables, à les faire concourir vers un but utile de quelque genre qu'il soit."

Such was the work and such the character of this wonderful man. Those whose desire to follow more closely and to witness the process of de-

velopment of the work of which this initial paper of Carnot was the introductory, should study the contribution of Sir William Thomson to this development, as published in 1849,—a paper which constitutes that physicist the virtual discoverer of Carnot and the godfather of the man and his thoughts. This paper constitutes the final chapter of this little book.

From that time the additional progress so rapidly made in the new science was as inevitable as the development of a gold-field, once the precious metal has been found in paying quantities in the hitherto unvisited cañons and gorges of a distant and unexplored mountain-range. But great as is the work since done, and great as have been the discoveries and the discoverers of later years, none claims our gratitude and compels our respect in greater degree than does the original discoverer—

SADI CARNOT.

II.

LIFE OF SADI CARNOT.

By M. H. Carnot.

As the life of Sadi Carnot was not marked by any notable event, his biography would have occupied only a few lines; but a scientific work by him, after remaining long in obscurity, brought again to light many years after his death, has caused his name to be placed among those of great inventors. In regard to his person, his mind, his character, nothing whatever has been known. Since there remains a witness of his private life—the sole witness, has he not a duty to fulfil? Ought he not to satisfy the natural and legitimate interest which attaches to any man whose work has deserved a portion of glory?

Nicolas-Léonard-Sadi Carnot was born June 1, 1796, in the smaller Luxembourg. This was that part of the palace where our father then dwelt as a member of the Directory. Our father had a predilection for the name of Sadi, which recalled to his mind ideas of wisdom and poetry. His first-born had borne this name, and despite the fate

of this poor child, who lived but a few months, he called the second also Sadi, in memory of the celebrated Persian poet and moralist.

Scarcely a year had passed when the proscription, which included the Director, obliged him to give up his life, or at least his liberty, to the conspirators of fructidor. Our mother carried her son far from the palace in which violation of law had just triumphed. She fled to St. Omer, with her family, while her husband was exiled to Switzerland, then to Germany.

Our mother often said to me, "Thy brother was born in the midst of the cares and agitations of grandeur, thou in the calm of an obscure retreat. Your constitutions show this difference of origin."

My brother in fact was of delicate constitution. He increased his strength later, by means of varied and judicious bodily exercises. He was of medium size, endowed with extreme sensibility and at the same time with extreme energy, more than reserved, almost rude, but singularly courageous on occasion. When he felt himself to be contending against injustice, nothing could restrain him. The following is an anecdote in illustration.

The Directory had given place to the Consulate. Carnot, after two years of exile, returned to his

country and was appointed Minister of War. Bonaparte at the same time was still in favor with the republicans. He remembered that Carnot had assisted him in the beginning of his military career, and he resumed the intimate relation which had existed between them during the Directory. When the minister went to Malmaison to work with the First Consul, he often took with him his son, then about four years old, to stay with Madame Bonaparte, who was greatly attached to him.

She was one day with some other ladies in a small boat on a pond, the ladies rowing the boat themselves, when Bonaparte, unexpectedly appearing, amused himself by picking up stones and throwing them near the boat, spattering water on the fresh toilets of the rowers. The ladies dared not manifest their displeasure, but the little Sadi, after having looked on at the affair for some time, suddenly placed himself boldly before the conqueror of Marengo, and threatening him with his fist, he cried "Beast of a First Consul, will you stop tormenting those ladies!"

Bonaparte, at this unexpected attack, stopped and looked in astonishment at the child. Then he was seized with a fit of laughter in which all the spectators of the scene joined.

At another time, when the minister, wishing to return to Paris, sought his son, who had been left with Madame Bonaparte, it was discovered that he had run away. They found him a long way off, in a mill, the mechanism of which he was trying to understand. This desire had been in the child's mind for days, and the honest miller, not knowing who he was, was kindly answering all his questions. Curiosity, especially in regard to mechanics and physics, was one of the essential traits of Sadi's mind.

On account of this disposition so early manifested, Carnot did not hesitate to give a scientific direction to the studies of his son. He was able to undertake this task himself when the monarchical tendencies of the new government had determined him to retire. For a few months only Sadi followed the course of M. Bourdon at the Charlemagne Lycée to prepare himself for the Polytechnic School.

The pupil made rapid progress. He was just sixteen years old when he was admitted to the school, the twenty-fourth on the list. This was in 1812. The following year he left it, first in artillery. But he was considered too young for the school of Metz, and he continued his studies at Paris for a year. To this circumstance is due the

fact that he took part in March, 1814, in the military exploits of Vincennes, and not of the butte Chaumont, as almost all the historians of the siege of Paris declared. M. Chasles, one of Sadi's school-fellows, took pains to rectify this error at a séance of the Institute in 1869.

If the pupils of the Polytechnic School did not earlier enter into the campaign, it was not because they had not asked to do so. I find in my brother's papers the copy of an address to the Emperor, signed by them December 29, 1813:

"SIRE: The country needs all its defenders. The pupils of the Polytechnic School, faithful to their motto, ask to be permitted to hasten to the frontiers to share the glory of the brave men who are consecrating themselves to the safety of France. The battalion, proud of having contributed to the defeat of the enemy, will return to the school to cultivate the sciences and prepare for new services."

General Carnot was at Anvers, which he had just been defending against the confederate English, Prussians, and Swedes, where the French flag yet floated, when he wrote to his son, April 12, 1814:

"MY DEAR SADI: I have learned with extreme pleasure that the battalion of the Polytechnic School has distinguished itself, and that you have performed your first military exploits with honor.

When I am recalled, I shall be very glad if the Minister of War will give you permission to come to me. You will become acquainted with a fine country and a beautiful city, where I have had the satisfaction of remaining in peace while disaster has overwhelmed so many other places."

Peace being restored, Sadi rejoined his father at Anvers and returned with him into France.

In the month of October he left the Polytechnic School, ranking sixth on the list of young men destined to service in the engineer corps, and went to Metz as a cadet sub-lieutenant at the school. Many scientific papers that he wrote there were a decided success. One is particularly referred to as very clever, a memoir on the instrument called the *theodolite* which is used in astronomy and geodesy.

I obtain these details from M. Ollivier, who was of the same rank as Sadi and who, later, was one of the founders of the *Ecole Centrale.* Among his other comrades besides M. Chasles, the learned geometrician just now referred to, was Gen. Duvivier, lamented victim of the insurrection of June 1848. I ought also to mention M. Robelin, Sadi's most intimate friend, who came to help me nurse him during his last illness, and who pub-

lished a notice concerning him in the *Revue encyclopédique,* t. lv.

The events of 1815 brought General Carnot back into politics during the "*Cent Jours*" which ended in a fresh catastrophe.

This gave Sadi a glimpse of human nature of which he could not speak without disgust. His little sub-lieutenant's room was visited by certain superior officers who did not disdain to mount to the third floor to pay their respects to the son of the new minister.

Waterloo put an end to their attentions. The Bourbons re-established on the throne, Carnot was proscribed and Sadi sent successively into many trying places to pursue his vocation of engineer, to count bricks, to repair walls, and to draw plans destined to be hidden in portfolios. He performed these duties conscientiously and without hope of recompense, for his name, which not long before had brought him so many flatteries, was henceforth the cause of his advancement being long delayed.

In 1818 there came an unlooked-for royal ordinance, authorizing the officers of all branches of the service to present themselves at the examinations for the new corps of the staff. Sadi was well aware that favor had much more to do with

this matter than ability, but he was weary of garrison life. The stay in small fortresses to which the nature of his work confined him did not offer sufficient resources to his love of study. Then he hoped, and his hope was realized, that a request for a furlough would be obtained without difficulty, and would insure him the leisure that he sought. In spite of the friendly opposition of some chiefs of the engineer corps, testifying to a sincere regret at the removal from their register of a name which had gained honor among them, Sadi came to Paris to take the examination, and was appointed lieutenant on the staff, January 20, 1819.

He hastened to obtain his furlough, and availed himself of it to lead, in Paris and in the country round about Paris, a studious life interrupted but once, in 1821, by a journey to Germany to visit our father in his exile at Magdeburg. We had then the pleasure of passing some weeks all three together.

When, two years later, death took from us this revered father and I returned alone to France, I found Sadi devoting himself to his scientific studies, which he alternated with the culture of the arts. In this way also, his tastes had marked out for him an original direction, for no one was more

opposed than he to the traditional and the conventional. On his music-desk were seen only the compositions of Lully that he had studied, and the concerti of Viotti which he executed. On his table were seen only Pascal, Molière, or La Fontaine, and he knew his favorite books almost by heart. I call this direction original, because it was anterior to the artistic and literary movement which preceded the revolution of 1830. As to the sympathy of Sadi for the author of the *Provinciales*, it was due not only to the respect of the young mathematician for one of the masters of science, but his devoutly religious mind regarded with horror hypocrisy and hypocrites.

Appreciating the useful and the beautiful, Sadi frequented the museum of the Louvre and the Italian Theatre, as well as the Jardin des Plantes and the Conservatoire des Arts et Metiers. Music was almost a passion with him. He probably inherited this from our mother, who was an excellent pianist, to whom Dalayrac and especially Monsigny, her compatriot, had given instruction. Not content with being able to play well on the violin, Sadi carried to great length his theoretical studies.

His insatiable intellect, moreover, would not allow him to remain a stranger to any branch of

knowledge. He diligently followed the course of the College of France and of the Sorbonne, of the École des Mines, of the Museum, and of the Bibliothèque. He visited the workshops with eager interest, and made himself familiar with the processes of manufacture; mathematical sciences, natural history, industrial art, political economy, —all these he cultivated with equal ardor. I have seen him not only practise as an amusement, but search theoretically into, gymnastics, fencing, swimming, dancing, and even skating. In even these things Sadi acquired a superiority which astonished specialists when by chance he forgot himself enough to speak of them, for the satisfaction of his own mind was the only aim that he sought.

He had such a repugnance to bringing himself forward that, in his intimate conversations with a few friends, he kept them ignorant of the treasures of science which he had accumulated. They never knew of more than a small part of them. How was it that he determined to formulate his ideas about the motive power of heat, and especially to publish them? I still ask myself this question,—I, who lived with him in the little apartment where our father was confined in the Rue du Parc-Royal while the police of the first Restoration were

threatening him. Anxious to be perfectly clear, Sadi made me read some passages of his manuscript in order to convince himself that it would be understood by persons occupied with other studies.

Perhaps a solitary life in small garrisons, in the work-room and in the chemical laboratory, had increased his natural reserve. In small companies, however, he was not at all taciturn. He took part voluntarily in the gayest plays, abandoning himself to lively chat. "The time passed in laughing is well spent," he once wrote. His language was at such times full of wit, keen without malice, original without eccentricity, sometimes paradoxical, but without other pretension than that of an innocent activity of intelligence. He had a very warm heart under a cold manner. He was obliging and devoted, sincere and true in his dealings.

Towards the end of 1826, a new royal ordinance having obliged the staff lieutenants to return to the ranks, Sadi asked and obtained a return to the engineer corps, in which he received the following year, as his rank of seniority, the grade of captain.

Military service, however, weighed upon him. Jealous of his liberty, in 1828, he laid aside his uniform that he might be free to come and go at

will. He took advantage of his leisure to make journeys and to visit our principal centres of industry.

He frequently visited M. Clement Desormes, professor at the *Conservatoire des Arts et Metiers,* who had made great advances in applied chemistry. M. Desormes willingly took counsel with him. He was a native of Bourgogne, our family country, which circumstance, I believe, brought them together.

It was before this period (in 1824) that Sadi had published his *Réflexions sur la puissance motrice du feu.* He had seen how little progress had been made in the theory of machines in which this power was employed. He had ascertained that the improvements made in their arrangement were effected tentatively, and almost by chance. He comprehended that in order to raise this important art above empiricism, and to give it the rank of a science, it was necessary to study the phenomena of the production of motion by heat, from the most general point of view, independently of any mechanism, of any special agent; and such had been the thought of his life.

Did he foresee that this small brochure would become the foundation of a new science? He must have attached much importance to it to

publish it, and bring himself out of his voluntary obscurity.

In fact (as his working notes prove), he perceived the existing relation between heat and mechanical work; and after having established the principle to which savants have given his name, he devoted himself to the researches which should enable him to establish with certainty the second principle, that of equivalence, which he already clearly divined. Thermodynamics was established from that time.

But these researches were rudely interrupted by a great event—the Revolution of July, 1830.

Sadi welcomed it enthusiastically—not, however, it is evident, as a personal advantage.

Several old members of the Convention were still living, even of those who had become celebrated; no favor of the new government was accorded them. To the son of Philippe-Egalité was ascribed a saying which, if it was untrue, at least agreed well with the sentiment of his position: "I can do nothing for the members of the Convention themselves," he said, "but for their families whatever they will."

However it may be, some of those about him vaguely questioned my brother as to his desires in case one of us should be called to the Chamber of

Peers, of which Carnot had been a member in 1815. We had on this occasion a brief conference. Unknown to us both, this distinction could be offered only to a title in some sort hereditary. We could not accept it without forsaking the principles of Carnot, who had combated the heredity of the peerage. The paternal opinion therefore came to second our distaste for the proposition, and dictated our reply.

Sadi frequented the popular reunions at this period without forsaking his *rôle* of a simple observer.

Nevertheless he was, when occasion demanded it, a man of prompt and energetic action. One incident will suffice to prove this, and to show the *sang-froid* which characterized him.

On the day of the funeral of Gen. Lamarque, Sadi was walking thoughtfully in the vicinity of the insurrection. A horseman preceding a company, and who was evidently intoxicated, passed along the street on the gallop, brandishing his sabre and striking down the passers-by. Sadi darted forward, cleverly avoided the weapon of the soldier, seized him by the leg, threw him to the earth and laid him in the gutter, then continued on his way to escape from the cheers of the crowd, amazed at this daring deed.

Before 1830, Sadi had formed part of a *Réunion polytechnique industrielle,* made up of old pupils of the school, with a plan of study in common. After 1830, he was a member of the *Association polytechnique,* consisting also of graduates, the object being the popular propagation of useful knowledge. The president of this association was M. de Choiseul-Praslin; the vice-presidents, MM. de Tracy, Auguste Comte, etc.

The hopes of the democracy meanwhile seeming to be in abeyance, Sadi devoted himself anew to study, and pursued his scientific labors with all the greater energy, as he brought to bear upon them the political ardor now so completely repressed. He undertook profound researches on the physical properties of gases and vapors, and especially on their elastic tensions. Unfortunately, the tables which he prepared from his comparative experiments were not completed; but happily the excellent works of Victor Regnault, so remarkable for their accuracy, have supplied to science, in this respect, the blanks of which Sadi Carnot was conscious.

His excessive application affected his health towards the end of June, 1832. Feeling temporarily better, he wrote gayly to one of his friends who had written several letters to him: "My delay this

time is not without excuse. I have been sick for a long time, and in a very wearisome way. I have had an inflammation of the lungs, followed by scarlet-fever. (Perhaps you know what this horrible disease is.) I had to remain twelve days in bed, without sleep or food, without any occupation, amusing myself with leeches, with drinks, with baths, and other toys out of the same shop. This little diversion is not yet ended, for I am still very feeble."

This letter was written at the end of July.

There was a relapse, then brain fever; then finally, hardly recovered from so many violent illnesses which had weakened him morally and physically, Sadi was carried off in a few hours, August 24, 1832, by an attack of cholera. Towards the last, and as if from a dark presentiment, he had given much attention to the prevailing epidemic, following its course with the attention and penetration that he gave to everything.

Sadi Carnot died in the vigor of life, in the brightness of a career that he bade fair to run with glory, leaving memory of profound esteem and affection in the hearts of many friends.

His copy-books, filled with memoranda, attest the activity of his mind, the variety of his knowledge, his love of humanity, his clear sentiments of

justice and of liberty. We can follow therein the traces of all his various studies. But the only work that he actually completed is this which is here published. It will suffice to preserve his name from oblivion.

His moral character has other claims on our recognition. Our only ambition here is to present a sketch of it. But, much better than through the perusal of these few pages, Sadi Carnot can be appreciated by reading the thoughts scattered through his memoranda, which are to be carefully collected. There are many practical rules of conduct which he records for himself; many observations that he desires to fix in his memory; sometimes an impression that has just come to him, grave or gay; sometimes too, though rarely, a trace of ill-humor directed against men or society. He never thought that these notes, the outpouring of his mind, would be read by other eyes than his own, or that they would some day be used to judge him. I find in them, for my part, touching analogies with the thoughts of my father, although the father and son had, unfortunately, lived almost always apart, by force of circumstances.*

* See the Appendix for these memoranda, and for other previously unpublished matter.

III.

REFLECTIONS ON THE MOTIVE-POWER OF HEAT, AND ON MACHINES FITTED TO DEVELOP THAT POWER.*

By S. Carnot.

Every one knows that heat can produce motion. That it possesses vast motive-power no one can doubt, in these days when the steam-engine is everywhere so well known.

To heat also are due the vast movements which take place on the earth. It causes the agitations of the atmosphere, the ascension of clouds, the fall of rain and of meteors, the currents of water which channel the surface of the globe, and of which

* Sadi Carnot's *Réflexions sur la puissance motrice du feu* (Paris, Bachelier 1824) was long ago completely exhausted. As but a small number of copies were printed, this remarkable work remained long unknown to the earlier writers on Thermodynamics. It was therefore for the benefit of savants unable to study a work out of print, as well as to render honor to the memory of Sadi Carnot, that the new publishers of the *Annales Scientifique de l'École Normale supérieure* (ii. series, t. 1, 1872) published a new edition, from which this translation is reproduced.

man has thus far employed but a small portion. Even earthquakes and volcanic eruptions are the result of heat.

From this immense reservoir we may draw the moving force necessary for our purposes. Nature, in providing us with combustibles on all sides, has given us the power to produce, at all times and in all places, heat and the impelling power which is the result of it. To develop this power, to appropriate it to our uses, is the object of heat-engines.

The study of these engines is of the greatest interest, their importance is enormous, their use is continually increasing, and they seem destined to produce a great revolution in the civilized world.

Already the steam-engine works our mines, impels our ships, excavates our ports and our rivers, forges iron, fashions wood, grinds grains, spins and weaves our cloths, transports the heaviest burdens, etc. It appears that it must some day serve as a universal motor, and be substituted for animal power, waterfalls, and air currents.

Over the first of these motors it has the advantage of economy, over the two others the inestimable advantage that it can be used at all times and places without interruption.

If, some day, the steam-engine shall be so per-

fected that it can be set up and supplied with fuel at small cost, it will combine all desirable qualities, and will afford to the industrial arts a range the extent of which can scarcely be predicted. It is not merely that a powerful and convenient motor that can be procured and carried anywhere is substituted for the motors already in use, but that it causes rapid extension in the arts in which it is applied, and can even create entirely new arts.

The most signal service that the steam-engine has rendered to England is undoubtedly the revival of the working of the coal-mines, which had declined, and threatened to cease entirely, in consequence of the continually increasing difficulty of drainage, and of raising the coal.* We should rank second the benefit to iron manufacture, both by the abundant supply of coal substituted for wood just when the latter had begun to grow scarce,

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*It may be said that coal-mining has increased tenfold in England since the invention of the steam-engine. It is almost equally true in regard to the mining of copper, tin, and iron. The results produced in a half-century by the steam-engine in the mines of England are to-day paralleled in the gold and silver mines of the New World—mines of which the working declined from day to day, principally on account of the insufficiency of the motors employed in the draining and the extraction of the minerals.

and by the powerful machines of all kinds, the use of which the introduction of the steam-engine has permitted or facilitated.

Iron and heat are, as we know, the supporters, the bases, of the mechanic arts. It is doubtful if there be in England a single industrial establishment of which the existence does not depend on the use of these agents, and which does not freely employ them. To take away to-day from England her steam-engines would be to take away at the same time her coal and iron. It would be to dry up all her sources of wealth, to ruin all on which her prosperity depends, in short, to annihilate that colossal power. The destruction of her navy, which she considers her strongest defence, would perhaps be less fatal.

The safe and rapid navigation by steamships may be regarded as an entirely new art due to the steam-engine. Already this art has permitted the establishment of prompt and regular communications across the arms of the sea, and on the great rivers of the old and new continents. It has made it possible to traverse savage regions where before we could scarcely penetrate. It has enabled us to carry the fruits of civilization over portions of the globe where they would else have been wanting for years. Steam navigation brings nearer together

the most distant nations. It tends to unite the nations of the earth as inhabitants of one country. In fact, to lessen the time, the fatigues, the uncertainties, and the dangers of travel—is not this the same as greatly to shorten distances?*

The discovery of the steam-engine owed its birth, like most human inventions, to rude attempts which have been attributed to different persons, while the real author is not certainly known. It is, however, less in the first attempts that the principal discovery consists, than in the successive improvements which have brought steam-engines to the condition in which we find them to-day. There is almost as great a distance between the first apparatus in which the expansive force of steam was displayed and the existing machine, as between the first raft that man ever made and the modern vessel.

If the honor of a discovery belongs to the nation in which it has acquired its growth and all its developments, this honor cannot be here refused

* We say, to lessen the dangers of journeys. In fact, although the use of the steam-engine on ships is attended by some danger which has been greatly exaggerated, this is more than compensated by the power of following always an appointed and well-known route, of resisting the force of the winds which would drive the ship towards the shore, the shoals, or the rocks.

to England. Savery, Newcomen, Smeaton, the famous Watt, Woolf, Trevithick, and some other English engineers, are the veritable creators of the steam-engine. It has acquired at their hands all its successive degrees of improvement. Finally, it is natural that an invention should have its birth and especially be developed, be perfected, in that place where its want is most strongly felt.

Notwithstanding the work of all kinds done by steam-engines, notwithstanding the satisfactory condition to which they have been brought to-day, their theory is very little understood, and the attempts to improve them are still directed almost by chance.

The question has often been raised whether the motive power of heat* is unbounded, whether the possible improvements in steam-engines have an assignable limit,—a limit which the nature of things will not allow to be passed by any means whatever; or whether, on the contrary, these improvements may be carried on indefinitely. We

* We use here the expression motive power to express the useful effect that a motor is capable of producing. This effect can always be likened to the elevation of a weight to a certain height. It has, as we know, as a measure, the product of the weight multiplied by the height to which it is raised.

have long sought, and are seeking to-day, to ascertain whether there are in existence agents preferable to the vapor of water for developing the motive power of heat; whether atmospheric air, for example, would not present in this respect great advantages. We propose now to submit these questions to a deliberate examination.

The phenomenon of the production of motion by heat has not been considered from a sufficiently general point of view. We have considered it only in machines the nature and mode of action of which have not allowed us to take in the whole extent of application of which it is susceptible. In such machines the phenomenon is, in a way, incomplete. It becomes difficult to recognize its principles and study its laws.

In order to consider in the most general way the principle of the production of motion by heat, it must be considered independently of any mechanism or any particular agent. It is necessary to establish principles applicable not only to steam-engines* but to all imaginable heat-engines, what-

* We distinguish here the steam-engine from the heat-engine in general. The latter may make use of any agent whatever, of the vapor of water or of any other, to develop the motive power of heat.

ever the working substance and whatever the method by which it is operated.

Machines which do not receive their motion from heat, those which have for a motor the force of men or of animals, a waterfall, an air-current, etc., can be studied even to their smallest details by the mechanical theory. All cases are foreseen, all imaginable movements are referred to these general principles, firmly established, and applicable under all circumstances. This is the character of a complete theory. A similar theory is evidently needed for heat-engines. We shall have it only when the laws of Physics shall be extended enough, generalized enough, to make known beforehand all the effects of heat acting in a determined manner on any body.

We will suppose in what follows at least a superficial knowledge of the different parts which compose an ordinary steam-engine; and we consider it unnecessary to explain what are the furnace, boiler, steam-cylinder, piston, condenser, etc.

The production of motion in steam-engines is always accompanied by a circumstance on which we should fix our attention. This circumstance is the re-establishing of equilibrium in the caloric; that is, its passage from a body in which the

temperature is more or less elevated, to another in which it is lower. What happens in fact in a steam-engine actually in motion? The caloric developed in the furnace by the effect of the combustion traverses the walls of the boiler, produces steam, and in some way incorporates itself with it. The latter carrying it away, takes it first into the cylinder, where it performs some function, and from thence into the condenser, where it is liquefied by contact with the cold water which it encounters there. Then, as a final result, the cold water of the condenser takes possession of the caloric developed by the combustion. It is heated by the intervention of the steam as if it had been placed directly over the furnace. The steam is here only a means of transporting the caloric. It fills the same office as in the heating of baths by steam, except that in this case its motion is rendered useful.

We easily recognize in the operations that we have just described the re-establishment of equilibrium in the caloric, its passage from a more or less heated body to a cooler one. The first of these bodies, in this case, is the heated air of the furnace; the second is the condensing water. The re-establishment of equilibrium of the caloric takes place between them, if not completely, at

least partially, for on the one hand the heated air, after having performed its function, having passed round the boiler, goes out through the chimney with a temperature much below that which it had acquired as the effect of combustion; and on the other hand, the water of the condenser, after having liquefied the steam, leaves the machine with a temperature higher than that with which it entered.

The production of motive power is then due in steam-engines not to an actual consumption of caloric, but *to its transportation from a warm body to a cold body*, that is, to its re-establishment of equilibrium—an equilibrium considered as destroyed by any cause whatever, by chemical action such as combustion, or by any other. We shall see shortly that this principle is applicable to any machine set in motion by heat.

According to this principle, the production of heat alone is not sufficient to give birth to the impelling power: it is necessary that there should also be cold; without it, the heat would be useless. And in fact, if we should find about us only bodies as hot as our furnaces, how can we condense steam? What should we do with it if once produced? We should not presume that we might discharge it into the atmosphere, as is done

in some engines;* the atmosphere would not receive it. It does receive it under the actual condition of things, only because it fulfils the office of a vast condenser, because it is at a lower temperature; otherwise it would soon become fully charged, or rather would be already saturated.†.

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* Certain engines at high pressure throw the steam out into the atmosphere instead of the condenser. They are used specially in places where it would be difficult to procure a stream of cold water sufficient to produce condensation.

† The existence of water in the liquid state here necessarily assumed, since without it the steam-engine could not be fed, supposes the existence of a pressure capable of preventing this water from vaporizing, consequently of a pressure equal or superior to the tension of vapor at that temperature. If such a pressure were not exerted by the atmospheric air, there would be instantly produced a quantity of steam sufficient to give rise to that tension, and it would be necessary always to overcome this pressure in order to throw out the steam from the engines into the new atmosphere. Now this is evidently equivalent to overcoming the tension which the steam retains after its condensation, as effected by ordinary means.

If a very high temperature existed at the surface of our globe, as it seems certain that it exists in its interior, all the waters of the ocean would be in a state of vapor in the atmosphere, and no portion of it would be found in a liquid state.

Wherever there exists a difference of temperature, wherever it has been possible for the equilibrium of the caloric to be re-established, it is possible to have also the production of impelling power. Steam is a means of realizing this power, but it is not the only one. All substances in nature can be employed for this purpose, all are susceptible of changes of volume, of successive contractions and dilatations, through the alternation of heat and cold. All are capable of overcoming in their changes of volume certain resistances, and of thus developing the impelling power. A solid body—a metallic bar for example—alternately heated and cooled increases and diminishes in length, and can move bodies fastened to its ends. A liquid alternately heated and cooled increases and diminishes in volume, and can overcome obstacles of greater or less size, opposed to its dilatation. An aeriform fluid is susceptible of considerable change of volume by variations of temperature. If it is enclosed in an expansible space, such as a cylinder provided with a piston, it will produce movements of great extent. Vapors of all substances capable of passing into a gaseous condition, as of alcohol, of mercury, of sulphur, etc., may fulfil the same office as vapor of water. The latter, alternately heated and cooled, would produce motive power in the shape

of permanent gases, that is, without ever returning to a liquid state. Most of these substances have been proposed, many even have been tried, although up to this time perhaps without remarkable success.

We have shown that in steam-engines the motive-power is due to a re-establishment of equilibrium in the caloric; this takes place not only for steam-engines, but also for every heat-engine—that is, for every machine of which caloric is the motor. Heat can evidently be a cause of motion only by virtue of the changes of volume or of form which it produces in bodies.

These changes are not caused by uniform temperature, but rather by alternations of heat and cold. Now to heat any substance whatever requires a body warmer than the one to be heated; to cool it requires a cooler body. We supply caloric to the first of these bodies that we may transmit it to the second by means of the intermediary substance. This is to re-establish, or at least to endeavor to re-establish, the equilibrium of the caloric.

It is natural to ask here this curious and important question: Is the motive power of heat invariable in quantity, or does it vary with the agent employed to realize it as the intermediary sub-

stance, selected as the subject of action of the heat?

It is clear that this question can be asked only in regard to a given quantity of caloric,* the difference of the temperatures also being given. We take, for example, one body A kept at a temperature of 100° and another body B kept at a temperature of 0°, and ask what quantity of motive power can be produced by the passage of a given portion of caloric (for example, as much as is necessary to melt a kilogram of ice) from the first of these bodies to the second. We inquire whether this quantity of motive power is necessarily limited, whether it varies with the substance employed to realize it, whether the vapor of water offers in this respect more or less advantage than the vapor of alcohol, of mercury, a permanent gas, or any other substance. We will try to answer these questions, availing ourselves of ideas already established.

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*It is considered unnecessary to explain here what is quantity of caloric or quantity of heat (for we employ these two expressions indifferently), or to describe how we measure these quantities by the calorimeter. Nor will we explain what is meant by latent heat, degree of temperature, specific heat, etc. The reader should be familiarized with these terms through the study of the elementary treatises of physics or of chemistry.

We have already remarked upon this self-evident fact, or fact which at least appears evident as soon as we reflect on the changes of volume occasioned by heat: *wherever there exists a difference of temperature, motive-power can be produced.* Reciprocally, wherever we can consume this power, it is possible to produce a difference of temperature, it is possible to occasion destruction of equilibrium in the caloric. Are not percussion and the friction of bodies actually means of raising their temperature, of making it reach spontaneously a higher degree than that of the surrounding bodies, and consequently of producing a destruction of equilibrium in the caloric, where equilibrium previously existed? It is a fact proved by experience, that the temperature of gaseous fluids is raised by compression and lowered by rarefaction. This is a sure method of changing the temperature of bodies, and destroying the equilibrium of the caloric as many times as may be desired with the same substance. The vapor of water employed in an inverse manner to that in which it is used in steam-engines can also be regarded as a means of destroying the equilibrium of the caloric. To be convinced of this we need but to observe closely the manner in which motive power is developed by the action of heat on vapor of water. Imagine

two bodies A and B, kept each at a constant temperature, that of A being higher than that of B. These two bodies, to which we can give or from which we can remove the heat without causing their temperatures to vary, exercise the functions of two unlimited reservoirs of caloric. We will call the first the furnace and the second the refrigerator.

If we wish to produce motive power by carrying a certain quantity of heat from the body A to the body B we shall proceed as follows :

(1) To borrow caloric from the body A to make steam with it—that is, to make this body fulfil the function of a furnace, or rather of the metal composing the boiler in ordinary engines—we here assume that the steam is produced at the same temperature as the body A.

(2) The steam having been received in a space capable of expansion, such as a cylinder furnished with a piston, to increase the volume of this space, and consequently also that of the steam. Thus rarefied, the temperature will fall spontaneously, as occurs with all elastic fluids ; admit that the rarefaction may be continued to the point where the temperature becomes precisely that of the body B.

(3) To condense the steam by putting it in contact with the body B, and at the same time exert-

ing on it a constant pressure until it is entirely liquefied. The body *B* fills here the place of the injection-water in ordinary engines, with this difference, that it condenses the vapor without mingling with it, and without changing its own temperature.*

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* We may perhaps wonder here that the body *B* being at the same temperature as the steam is able to condense it. Doubtless this is not strictly possible, but the slightest difference of temperature will determine the condensation, which suffices to establish the justice of our reasoning. It is thus that, in the differential calculus, it is sufficient that we can conceive the neglected quantities indefinitely reducible in proportion to the quantities retained in the equations, to make certain of the exact result.

The body *B* condenses the steam without changing its own temperature—this results from our supposition. We have admitted that this body may be maintained at a constant temperature. We take away the caloric as the steam furnishes it. This is the condition in which the metal of the condenser is found when the liquefaction of the steam is accomplished by applying cold water externally, as was formerly done in several engines. Similarly, the water of a reservoir can be maintained at a constant level if the liquid flows out at one side as it flows in at the other.

One could even conceive the bodies *A* and *B* maintaining the same temperature, although they might lose or gain certain quantities of heat. If, for example, the body *A* were a mass of steam ready to become liquid, and the body

The operations which we have just described might have been performed in an inverse direction and order. There is nothing to prevent forming vapor with the caloric of the body *B*, and at the temperature of that body, compressing it in such a way as to make it acquire the temperature of the body *A*, finally condensing it by contact with this latter body, and continuing the compression to complete liquefaction.

By our first operations there would have been at the same time production of motive power and transfer of caloric from the body *A* to the body *B*. By the inverse operations there is at the same time expenditure of motive power and return of caloric from the body *B* to the body *A*. But if we have acted in each case on the same quantity of vapor, if there is produced no loss either of motive power or caloric, the quantity of motive power produced in the first place will be equal to that which would have been expended in the second, and the quantity of caloric passed in the first case from the body *A* to the body *B* would be equal to the quantity which passes back again in the second from the body *B* to the body *A*; so that an indefi-

B a mass of ice ready to melt, these bodies might, as we know, furnish or receive caloric without thermometric change.

nite number of alternative operations of this sort could be carried on without in the end having either produced motive power or transferred caloric from one body to the other.

Now if there existed any means of using heat preferable to those which we have employed, that is, if it were possible by any method whatever to make the caloric produce a quantity of motive power greater than we have made it produce by our first series of operations, it would suffice to divert a portion of this power in order by the method just indicated to make the caloric of the body *B* return to the body *A* from the refrigerator to the furnace, to restore the initial conditions, and thus to be ready to commence again an operation precisely similar to the former, and so on: this would be not only perpetual motion, but an unlimited creation of motive power without consumption either of caloric or of any other agent whatever. Such a creation is entirely contrary to ideas now accepted, to the laws of mechanics and of sound physics. It is inadmissible.* We should then conclude that *the maximum of motive power resulting from the employment of steam is also the maximum of motive power realizable by any means whatever.* We will

* Note A, Appendix B.

soon give a second more vigorous demonstration of this theory. This should be considered only as an approximation. (See page 59.)

We have a right to ask, in regard to the proposition just enunciated, the following questions: What is the sense of the word *maximum* here? By what sign can it be known that this maximum is attained? By what sign can it be known whether the steam is employed to greatest possible advantage in the production of motive power?

Since every re-establishment of equilibrium in the caloric may be the cause of the production of motive power, every re-establishment of equilibrium which shall be accomplished without production of this power should be considered as an actual loss. Now, very little reflection would show that all change of temperature which is not due to a change of volume of the bodies can be only a useless re-establishment of equilibrium in the caloric.* The necessary condition of the maximum is, then, *that*

* We assume here no chemical action between the bodies employed to realize the motive power of heat. The chemical action which takes place in the furnace is, in some sort, a preliminary action,—an operation destined not to produce immediately motive power, but to destroy the equilibrium of the caloric, to produce a difference of temperature which may finally give rise to motion.

in the bodies employed to realize the motive power of heat there should not occur any change of temperature which may not be due to a change of volume. Reciprocally, every time that this condition is fulfilled the maximum will be attained. This principle should never be lost sight of in the construction of heat-engines; it is its fundamental basis. If it cannot be strictly observed, it should at least be departed from as little as possible.

Every change of temperature which is not due to a change of volume or to chemical action (an action that we provisionally suppose not to occur here) is necessarily due to the direct passage of the caloric from a more or less heated body to a colder body. This passage occurs mainly by the contact of bodies of different temperatures; hence such contact should be avoided as much as possible. It cannot probably be avoided entirely, but it should at least be so managed that the bodies brought in contact with each other differ as little as possible in temperature. When we just now supposed, in our demonstration, the caloric of the body *A* employed to form steam, this steam was considered as generated at the temperature of the body *A*; thus the contact took place only between bodies of equal temperatures; the change of temperature occurring afterwards in the steam was due to dilatation, con-

sequently to a change of volume. Finally, condensation took place also without contact of bodies of different temperatures. It occurred while exerting a constant pressure on the steam brought in contact with the body *B* of the same temperature as itself. The conditions for a maximum are thus found to be fulfilled. In reality the operation cannot proceed exactly as we have assumed. To determine the passage of caloric from one body to another, it is necessary that there should be an excess of temperature in the first, but this excess may be supposed as slight as we please. We can regard it as insensible in theory, without thereby destroying the exactness of the arguments.

A more substantial objection may be made to our demonstration, thus: When we borrow caloric from the body *A* to produce steam, and when this steam is afterwards condensed by its contact with the body *B*, the water used to form it, and which we considered at first as being of the temperature of the body *A*, is found at the close of the operation at the temperature of the body *B*. It has become cool. If we wish to begin again an operation similar to the first, if we wish to develop a new quantity of motive power with the same instrument, with the same steam, it is necessary first to re-establish the original condition—to restore

the water to the original temperature. This can undoubtedly be done by at once putting it again in contact with the body *A*; but there is then contact between bodies of different temperatures, and loss of motive power.* It would be impossible to execute the inverse operation, that is, to return to the body *A* the caloric employed to raise the temperature of the liquid.

This difficulty may be removed by supposing the difference of temperature between the body *A* and the body *B* indefinitely small. The quantity of heat necessary to raise the liquid to its former

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* This kind of loss is found in all steam-engines. In fact, the water destined to feed the boiler is always cooler than the water which it already contains. There occurs between them a useless re-establishment of equilibrium of caloric. We are easily convinced, *à posteriori*, that this re-establishment of equilibrium causes a loss of motive power if we reflect that it would have been possible to previously heat the feed-water by using it as condensing-water in a small accessory engine, when the steam drawn from the large boiler might have been used, and where the condensation might be produced at a temperature intermediate between that of the boiler and that of the principal condenser. The power produced by the small engine would have cost no loss of heat, since all that which had been used would have returned into the boiler with the water of condensation.

temperature will be also indefinitely small and unimportant relatively to that which is necessary to produce steam—a quantity always limited.

The proposition found elsewhere demonstrated for the case in which the difference between the temperatures of the two bodies is indefinitely small, may be easily extended to the general case. In fact, if it operated to produce motive power by the passage of caloric from the body A to the body Z, the temperature of this latter body being very different from that of the former, we should imagine a series of bodies B, C, D . . . of temperatures intermediate between those of the bodies A, Z, and selected so that the differences from A to B, from B to C, etc., may all be indefinitely small. The caloric coming from A would not arrive at Z till after it had passed through the bodies B, C, D, etc., and after having developed in each of these stages maximum motive power. The inverse operations would here be entirely possible, and the reasoning of page 52 would be strictly applicable.

According to established principles at the present time, we can compare with sufficient accuracy the motive power of heat to that of a waterfall. Each has a maximum that we cannot exceed, whatever may be, on the one hand, the machine which is acted upon by the water, and whatever, on the

other hand, the substance acted upon by the heat. The motive power of a waterfall depends on its height and on the quantity of the liquid; the motive power of heat depends also on the quantity of caloric used, and on what may be termed, on what in fact we will call, the *height of its fall*,* that is to say, the difference of temperature of the bodies between which the exchange of caloric is made. In the waterfall the motive power is exactly proportional to the difference of level between the higher and lower reservoirs. In the fall of caloric the motive power undoubtedly increases with the difference of temperature between the warm and the cold bodies; but we do not know whether it is proportional to this difference. We do not know, for example, whether the fall of caloric from 100 to 50 degrees furnishes more or less motive power than the fall of this same caloric from 50 to zero. It is a question which we propose to examine hereafter.

We shall give here a second demonstration of the fundamental proposition enunciated on page 56, and present this proposition under a more general form than the one already given.

* The matter here dealt with being entirely new, we are obliged to employ expressions not in use as yet, and which perhaps are less clear than is desirable.

When a gaseous fluid is rapidly compressed its temperature rises. It falls, on the contrary, when it is rapidly dilated. This is one of the facts best demonstrated by experiment. We will take it for the basis of our demonstration.*

If, when the temperature of a gas has been raised by compression, we wish to reduce it to its former temperature without subjecting its volume to new changes, some of its caloric must be removed. This caloric might have been removed in proportion as pressure was applied, so that the temperature of the gas would remain constant. Similarly, if the gas is rarefied we can avoid lowering the temperature by supplying it with a certain quantity of caloric. Let us call the caloric employed at such times, when no change of temperature occurs, *caloric due to change of volume.* This denomination does not indicate that the caloric appertains to the volume : it does not appertain to it any more than to pressure, and might as well be called *caloric due to the change of pressure.* We do not know what laws it follows relative to the variations of volume : it is possible that its quantity changes either with the nature of the gas, its density, or its temperature. Ex-

* Note 13, Appendix B.

periment has taught us nothing on this subject. It has only shown us that this caloric is developed in greater or less quantity by the compression of the elastic fluids.

This preliminary idea being established, let us imagine an elastic fluid, atmospheric air for example, shut up in a cylindrical vessel, *abcd* (Fig. 1), provided with a movable diaphragm or piston, *cd*. Let there be also two bodies, *A* and *B*, kept each at a constant temperature, that of *A* being higher than that of *B*. Let us picture to ourselves now the series of operations which are to be described :

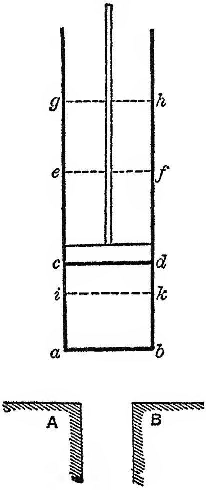

FIG. 1

(1) Contact of the body *A* with the air enclosed in the space *abcd* or with the wall of this space—a wall that we will suppose to transmit the caloric readily. The air becomes by such contact of the same temperature as the body *A*; *cd* is the actual position of the piston.

(2) The piston gradually rises and takes the position *ef*. The body *A* is all the time in con-

tact with the air, which is thus kept at a constant temperature during the rarefaction. The body *A* furnishes the caloric necessary to keep the temperature constant.

(3) The body *A* is removed, and the air is then no longer in contact with any body capable of furnishing it with caloric. The piston meanwhile continues to move, and passes from the position *ef* to the position *gh*. The air is rarefied without receiving caloric, and its temperature falls. Let us imagine that it falls thus till it becomes equal to that of the body *B*; at this instant the piston stops, remaining at the position *gh*.

(4) The air is placed in contact with the body *B*; it is compressed by the return of the piston as it is moved from the position *gh* to the position *cd*. This air remains, however, at a constant temperature because of its contact with the body *B*, to which it yields its caloric.

(5) The body *B* is removed, and the compression of the air is continued, which being then isolated, its temperature rises. The compression is continued till the air acquires the temperature of the body *A*. The piston passes during this time from the position *cd* to the position *ik*.

(6) The air is again placed in contact with the body *A*. The piston returns from the position *ik*

to the position *ef*; the temperature remains unchanged.

(7) The step described under number 3 is renewed, then successively the steps 4, 5, 6, 3, 4, 5, 6, 3, 4, 5; and so on.

In these various operations the piston is subject to an effort of greater or less magnitude, exerted by the air enclosed in the cylinder; the elastic force of this air varies as much by reason of the changes in volume as of changes of temperature. But it should be remarked that with equal volumes, that is, for the similar positions of the piston, the temperature is higher during the movements of dilatation than during the movements of compression. During the former the elastic force of the air is found to be greater, and consequently the quantity of motive power produced by the movements of dilatation is more considerable than that consumed to produce the movements of compression. Thus we should obtain an excess of motive power—an excess which we could employ for any purpose whatever. The air, then, has served as a heat-engine; we have, in fact, employed it in the most advantageous manner possible, for no useless re-establishment of equilibrium has been effected in the caloric.

All the above-described operations may be

executed in an inverse sense and order. Let us imagine that, after the sixth period, that is to say the piston having arrived at the position *ef*, we cause it to return to the position *ik*, and that at the same time we keep the air in contact with the body *A*. The caloric furnished by this body during the sixth period would return to its source, that is, to the body *A*, and the conditions would then become precisely the same as they were at the end of the fifth period. If now we take away the body *A*, and if we cause the piston to move from *ef* to *cd*, the temperature of the air will diminish as many degrees as it increased during the fifth period, and will become that of the body *B*. We may evidently continue a series of operations the inverse of those already described. It is only necessary under the same circumstances to execute for each period a movement of dilatation instead of a movement of compression, and reciprocally.

The result of these first operations has been the production of a certain quantity of motive power and the removal of caloric from the body *A* to the body *B*. The result of the inverse operations is the consumption of the motive power produced and the return of the caloric from the body *B* to the body *A*; so that these two series of operations annul

each other, after a fashion, one neutralizing the other.

The impossibility of making the caloric produce a greater quantity of motive power than that which we obtained from it by our first series of operations, is now easily proved. It is demonstrated by reasoning very similar to that employed at page 56; the reasoning will here be even more exact. The air which we have used to develop the motive power is restored at the end of each cycle of operations exactly to the state in which it was at first found, while, as we have already remarked, this would not be precisely the case with the vapor of water.*

* We tacitly assume in our demonstration, that when a body has experienced any changes, and when after a certain number of transformations it returns to precisely its original state, that is, to that state considered in respect to density, to temperature, to mode of aggregation—let us suppose, I say, that this body is found to contain the same quantity of heat that it contained at first, or else that the quantities of heat absorbed or set free in these different transformations are exactly compensated. This fact has never been called in question. It was first admitted without reflection, and verified afterwards in many cases by experiments with the calorimeter. To deny it would be to overthrow the whole theory of heat to which it serves as a basis. For the rest, we may say in passing, the main

We have chosen atmospheric air as the instrument which should develop the motive power of heat, but it is evident that the reasoning would have been the same for all other gaseous substances, and even for all other bodies susceptible of change of temperature through successive contractions and dilatations, which comprehends all natural substances, or at least all those which are adapted to realize the motive power of heat. Thus we are led to establish this general proposition:

The motive power of heat is independent of the agents employed to realize it; its quantity is fixed solely by the temperatures of the bodies between which is effected, finally, the transfer of the caloric.

We must understand here that each of the methods of developing motive power attains the perfection of which it is susceptible. This condition is found to be fulfilled if, as we remarked above, there is produced in the body no other change of temperature than that due to change of volume, or, what is the same thing in other words, if there is no contact between bodies of sensibly different temperatures.

Different methods of realizing motive power may

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principles on which the theory of heat rests require the most careful examination. Many experimental facts appear almost inexplicable in the present state of this theory.

be taken, as in the employment of different substances, or in the use of the same substance in two different states—for example, of a gas at two different densities.

This leads us naturally to those interesting researches on the aeriform fluids—researches which lead us also to new results in regard to the motive power of heat, and give us the means of verifying, in some particular cases, the fundamental proposition above stated.*

We readily see that our demonstration would have been simplified by supposing the temperatures of the bodies *A* and *B* to differ very little. Then the movements of the piston being slight during the periods 3 and 5, these periods might have been suppressed without influencing sensibly the production of motive power. A very little change of volume should suffice in fact to produce a very slight change of temperature, and this slight change of volume may be neglected in presence of that of the periods 4 and 6, of which the extent is unlimited.

If we suppress periods 3 and 5, in the series of

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* We will suppose, in what follows, the reader to be *au courant* with the later progress of modern Physics in regard to gaseous substances and heat.

operations above described, it is reduced to the following:

(1) Contact of the gas confined in *abcd* (Fig. 2) with the body *A*, passage of the piston from *cd* to *ef*.

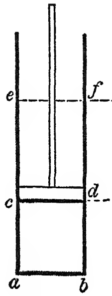

FIG. 2.

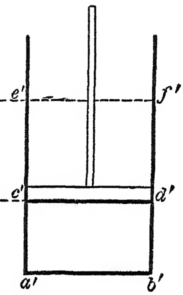

FIG. 3.

(2) Removal of the body *A*, contact of the gas confined in *abef* with the body *B*, return of the piston from *ef* to *cd*.

(3) Removal of the body *B*, contact of the gas with the body *A*, passage of the piston from *cd* to *ef*, that is, repetition of the first period, and so on.

The motive power resulting from the *ensemble* of operations 1 and 2 will evidently be the difference between that which is produced by the expansion of the gas while it is at the temperature of the body *A*, and that which is consumed to compress this gas while it is at the temperature of the body *B*.

Let us suppose that operations 1 and 2 be performed on two gases of different chemical natures but under the same pressure—under atmospheric pressure, for example. These two gases will behave exactly alike under the same circumstances, that is, their expansive forces, originally equal, will remain always equal, whatever may be the variations of volume and of temperature, provided these variations are the same in both. This results obviously from the laws of Mariotte and MM. Gay-Lussac and Dalton—laws common to all elastic fluids, and in virtue of which the same relations exist for all these fluids between the volume, the expansive force, and the temperature.

Since two different gases at the same temperature and under the same pressure should behave alike under the same circumstances, if we subjected them both to the operations above described, they should give rise to equal quantities of motive power.

Now this implies, according to the fundamental proposition that we have established, the employment of two equal quantities of caloric; that is, it implies that the quantity of caloric transferred from the body *A* to the body *B* is the same, whichever gas is used.

The quantity of caloric transferred from the body *A* to the body *B* is evidently that which is

absorbed by the gas in its expansion of volume, or that which this gas relinquishes during compression. We are led, then, to establish the following proposition :

When a gas passes without change of temperature from one definite volume and pressure to another volume and another pressure equally definite, the quantity of caloric absorbed or relinquished is always the same, whatever may be the nature of the gas chosen as the subject of the experiment.

Take, for example, 1 litre of air at the temperature of 100° and under the pressure of one atmosphere. If we double the volume of this air and wish to maintain it at the temperature of 100°, a certain quantity of heat must be supplied to it. Now this quantity will be precisely the same if, instead of operating on the air, we operate upon carbonic-acid gas, upon nitrogen, upon hydrogen, upon vapor of water or of alcohol, that is, if we double the volume of 1 litre of these gases taken at the temperature of 100° and under atmospheric pressure.

It will be the same thing in the inverse sense if, instead of doubling the volume of gas, we reduce it one half by compression. The quantity of heat that the elastic fluids set free or absorb in their changes of volume has never been measured by

any direct experiment, and doubtless such an experiment would be very difficult, but there exists a datum which is very nearly its equivalent. This has been furnished by the theory of sound. It deserves much confidence because of the exactness of the conditions which have led to its establishment. It consists in this:

Atmospheric air should rise one degree Centigrade when by sudden compression it experiences a reduction of volume of $\frac{1}{116}$.*

Experiments on the velocity of sound having been made in air under the pressure of 760 millimetres of mercury and at the temperature of 6°, it is only to these two circumstances that our datum has reference. We will, however, for greater facility, refer it to the temperature 0°, which is nearly the same.

Air compressed $\frac{1}{116}$, and thus heated one degree, differs from air heated directly one degree only in its density. The primitive volume being supposed

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* M. Poisson, to whom this figure is due, has shown that it accords very well with the result of an experiment of MM. Clement and Desormes on the return of air into a vacuum, or rather, into air slightly rarefied. It also accords very nearly with results found by MM. Gay-Lussac and Welter. (See note, p. 87.)

to be V, the compression of $\frac{1}{116}$ reduces it to $V - \frac{1}{116}V$.

Direct heating under constant pressure should, according to the rule of M. Gay-Lussac, increase the volume of air $\frac{1}{267}$ above what it would be at $0°$: so the air is, on the one hand, reduced to the volume $V - \frac{1}{116}V$; on the other, it is increased to $V + \frac{1}{267}V$.

The difference between the quantities of heat which the air possesses in both cases is evidently the quantity employed to raise it directly one degree; so then the quantity of heat that the air would absorb in passing from the volume $V - \frac{1}{116}V$ to the volume $V + \frac{1}{267}V$ is equal to that which is required to raise it one degree.

Let us suppose now that, instead of heating one degree the air subjected to a constant pressure and able to dilate freely, we inclose it within an invariable space, and that in this condition we cause it to rise one degree in temperature. The air thus heated one degree will differ from the air compressed $\frac{1}{116}$ only by its $\frac{1}{116}$ greater volume. So then the quantity of heat that the air would set free by a reduction of volume of $\frac{1}{116}$ is equal to that which would be required to raise it one degree Centigrade under constant volume. As the differences between the volumes $V - \frac{1}{116}V$, V, and

$V + \frac{1}{267}V$ are small relatively to the volumes themselves, we may regard the quantities of heat absorbed by the air in passing from the first of these volumes to the second, and from the first to the third, as sensibly proportional to the changes of volume. We are then led to the establishment of the following relation:

The quantity of heat necessary to raise one degree air under constant pressure is to the quantity of heat necessary to raise one degree the same air under constant volume, in the ratio of the numbers

$$\frac{1}{116} + \frac{1}{267} \quad \text{to} \quad \frac{1}{116};$$

or, multiplying both by 116×267, in the ratio of the numbers $267 + 116$ to 267.

This, then, is the ratio which exists between the capacity of air for heat under constant pressure and its capacity under constant volume. If the first of these two capacities is expressed by unity, the other will be expressed by the number $\frac{267}{267 + 116}$, or very nearly 0.700; their difference, $1 - 0.700$ or 0.300, will evidently express the quantity of heat which will produce the increase of volume in the air when it is heated one degree under constant pressure.

According to the law of MM. Gay-Lussac and Dalton, this increase of volume would be the same

for all other gases; according to the theory demonstrated on page 87, the heat absorbed by these equal increases of volume is the same for all the elastic fluids, which leads to the establishment of the following proposition:

The difference between specific heat under constant pressure and specific heat under constant volume is the same for all gases.

It should be remarked here that all the gases are considered as taken under the same pressure, atmospheric pressure for example, and that the specific heats are also measured with reference to the volumes.

It is a very easy matter now for us to prepare a table of the specific heat of gases under constant volume, from the knowledge of their specific heats under constant pressure. Here is the table:

TABLE OF THE SPECIFIC HEAT OF GASES.

NAMES OF GASES.	Specific Heat under Const. Press.	Specific Heat at Const. Vol.
Atmospheric Air, ...	1.000	0.700
Hydrogen Gas, ...	0.903	0.603
Carbonic Acid, ...	1.258	0.958
Oxygen, ...	0.976	0.676
Nitrogen, ...	1.000	0.700
Protoxide of Nitrogen, . .	1.350	1.050
Olefiant Gas, ...	1.553	1.253
Oxide of Carbon, ...	1.034	0.734

The first column is the result of the direct experiments of MM. Delaroche and Bérard on the specific heat of the gas under atmospheric pressure, and the second column is composed of the numbers of the first diminished by 0.300.

The numbers of the first column and those of the second are here referred to the same unit, to the specific heat of atmospheric air under constant pressure.

The difference between each number of the first column and the corresponding number of the second being constant, the relation between these numbers should be variable. Thus the ~~relation~~ between the specific heat of gases under constant pressure and the specific heat at constant volume, varies in different gases.

We have seen that air when it is subjected to a sudden compression of $\frac{1}{116}$ of its volume rises one degree in temperature. The other gases through a similar compression should also rise in temperature. They should rise, but not equally, in inverse ratio with their specific heat at constant volume. In fact, the reduction of volume being by hypothesis always the same, the quantity of heat due to this reduction should likewise be always the same, and consequently should produce an elevation of temperature dependent only on the specific heat

acquired by the gas after its compression, and evidently in inverse ratio with this specific heat. Thus we can easily form the table of the elevations of temperature of the different gases for a compression of $\frac{1}{116}$.

TABLE OF THE ELEVATION OF TEMPERATURE

OF

Gases through the Effect of Compression.

NAMES OF GASES.	Elevation of Temperature for a Reduction of Volume of $\frac{1}{116}$.
	°
Atmospheric Air, ...	1.000
Hydrogen Gas, ...	1.160
Carbonic Acid, ...	0.730
Oxygen, ...	1.035
Nitrogen, ...	1.000
Protoxide of Nitrogen, . . .	0.667
Olefiant Gas, ...	0.558
Carbonic Oxide, ...	0.955

A second compression of $\frac{1}{116}$ (of the altered volume), as we shall presently see, would also raise the temperature of these gases nearly as much as the first; but it would not be the same with a third, a fourth, a hundredth such compression. The capacity of gases for heat changes with their volume. It is not unlikely that it changes also with the temperature.

We shall now deduce from the general proposi-

tion stated on page 68 a second theory, which will serve as a corollary to that just demonstrated.

Let us suppose that the gas enclosed in the cylindrical space *abcd* (Fig. 2) be transported into the space *a′b′c′d′* (Fig. 3) of equal height, but of different base and wider. This gas would increase in volume, would diminish in density and in elastic force, in the inverse ratio of the two volumes *abcd*, *a′b′c′d′*. As to the total pressure exerted in each piston *cd*, *c′d′*, it would be the same from all quarters, for the surface of these pistons is in direct ratio to the volumes.

Let us suppose that we perform on the gas inclosed in *a′b′c′d′* the operations described on page 70, and which were taken as having been performed upon the gas inclosed in *abcd*; that is, let us suppose that we have given to the piston *c′d′* motions equal to those of the piston *cd*, that we have made it occupy successively the positions *c′d′* corresponding to *cd*, and *e′f′* corresponding to *ef*, and that at the same time we have subjected the gas by means of the two bodies *A* and *B* to the same variations of temperature as when it was inclosed in *abcd* The total effort exercised on the piston would be found to be, in the two cases, always the same at the corresponding instants. This results solely from

the law of Mariotte.* In fact, the densities of the two gases maintaining always the same ratio for similar positions of the pistons, and the temperatures being always equal in both, the total pressures exercised on the pistons will always maintain the same ratio to each other. If this ratio is, at any instant whatever, unity, the pressures will always be equal.

As, furthermore, the movements of the two pistons have equal extent, the motive power produced by each will evidently be the same; whence we should conclude, according to the proposition on

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* The law of Mariotte, which is here made the foundation upon which to establish our demonstration, is one of the best authenticated physical laws. It has served as a basis to many theories verified by experience, and which in turn verify all the laws on which they are founded. We can cite also, as a valuable verification of Mariotte's law and also of that of MM. Gay-Lussac and Dalton, for a great difference of temperature, the experiments of MM. Dulong and Petit. (See *Annales de Chimie et de Physique*, Feb. 1818, t. vii. p. 122.)

The more recent experiments of Davy and Faraday can also be cited.

The theories that we deduce here would not perhaps be exact if applied outside of certain limits either of density or temperature. They should be regarded as true only within the limits in which the laws of Mariotte and of MM. Gay-Lussac and Dalton are themselves proven.

page 68, that the quantities of heat consumed by each are the same, that is, that there passes from the body A to the body B the same quantity of heat in both cases.

The heat abstracted from the body A and communicated to the body B, is simply the heat absorbed during the rarefaction of the gas, and afterwards liberated by its compression. We are therefore led to establish the following theorem:

When an elastic fluid passes without change of temperature from the volume U to the volume V, and when a similar ponderable quantity of the same gas passes at the same temperature from the volume U' to the volume V', if the ratio of U' to V' is found to be the same as the ratio of U to V, the quantities of heat absorbed or disengaged in the two cases will be equal.

This theorem might also be expressed as follows:

When a gas varies in volume without change of temperature, the quantities of heat absorbed or liberated by this gas are in arithmetical progression, if the increments or the decrements of volume are found to be in geometrical progression.

When a litre of air maintained at a temperature of ten degrees is compressed, and when it is reduced to one half a litre, a certain quantity of heat is set free. This quantity will be found always

the same if the volume is further reduced from a half litre to a quarter litre, from a quarter litre to an eighth, and so on.

If, instead of compressing the air, we carry it successively to two litres, four litres, eight litres, etc., it will be necessary to supply to it always equal quantities of heat in order to maintain a constant temperature.

This readily accounts for the high temperature attained by air when rapidly compressed. We know that this temperature inflames tinder and even makes air luminous. If, for a moment, we suppose the specific heat of air to be constant, in spite of the changes of volume and temperature, the temperature will increase in arithmetical progression for reduction of volume in geometrical progression.

Starting from this datum, and admitting that one degree of elevation in the temperature corresponds to a compression of $\frac{1}{116}$, we shall readily come to the conclusion that air reduced to $\frac{1}{14}$ of its primitive volume should rise in temperature about 300 degrees, which is sufficient to inflame tinder.*

* When the volume is reduced $\frac{1}{116}$, that is, when it becomes $\frac{115}{116}$ of what it was at first, the temperature rises one degree. Another reduction of $\frac{1}{116}$ carries the volume

The elevation of temperature ought, evidently, to be still more considerable if the capacity of the air for heat becomes less as its volume diminishes. Now this is probable, and it also seems to follow from the experiments of MM. Delaroche and Bérard on the specific heat of air taken at different densities. (See the Mémoire in the *Annales de Chimie*, t. lxxxv. pp. 72, 224.)

The two theorems explained on pp. 72 and 81 suffice for the comparison of the quantities of heat absorbed or set free in the changes of volume of elastic fluids, whatever may be the density and the chemical nature of these fluids, provided always

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to $(\frac{115}{116})^2$, and the temperature should rise another degree. After x similar reductions the volume becomes $(\frac{115}{116})^x$, and the temperature should be raised x degrees. If we suppose $(\frac{115}{116})^x = \frac{1}{14}$, and if we take the logarithms of both, we find

$$x = \text{about } 300^\circ.$$

If we suppose $(\frac{115}{116})^x = \frac{1}{2}$, we find

$$x = 80^\circ;$$

which shows that air compressed one half rises 80°.

All this is subject to the hypothesis that the specific heat of air does not change, although the volume diminishes. But if, for the reasons hereafter given (pp. 86, 89), we regard the specific heat of air compressed one half as reduced in the relation of 700 to 616, the number 80° must be multiplied by $\frac{700}{616}$, which raises it to 90°.

that they be taken and maintained at a certain invariable temperature. But these theories furnish no means of comparing the quantities of heat liberated or absorbed by elastic fluids which change in volume at different temperatures. Thus we are ignorant what relation exists between the heat relinquished by a litre of air reduced one half, the temperature being kept at zero, and the heat relinquished by the same litre of air reduced one half, the temperature being kept at 100°. The knowledge of this relation is closely connected with that of the specific heat of gases at various temperatures, and to some other data that Physics as yet does not supply.

The second of our theorems offers us a means of determining according to what law the specific heat of gases varies with their density.

Let us suppose that the operations described on p. 70, instead of being performed with two bodies, *A*, *B*, of temperatures differing indefinitely small, were carried on with two bodies whose temperatures differ by a finite quantity—one degree, for example. In a complete circle of operations the body *A* furnishes to the elastic fluid a certain quantity of heat, which may be divided into two portions: (1) That which is necessary to maintain the temperature of the fluid constant during dilata-

tion; (2) that which is necessary to restore the temperature of the fluid from that of the body B to that of the body A, when, after having brought back this fluid to its primitive volume, we place it again in contact with the body A. Let us call the first of these quantities a and the second b. The total caloric furnished by the body A will be expressed by $a + b$.

The caloric transmitted by the fluid to the body B may also be divided into two parts: one, b', due to the cooling of the gas by the body B; the other, a', which the gas abandons as a result of its reduction of volume. The sum of these two quantities is $a' + b'$; it should be equal to $a + b$, for, after a complete cycle of operations, the gas is brought back exactly to its primitive state. It has been obliged to give up all the caloric which has first been furnished to it. We have then

$$a + b = a' + b';$$

or rather,

$$a - a' = b' - b.$$

Now, according to the theorem given on page 81, the quantities a and a' are independent of the density of the gas, provided always that the ponderable quantity remains the same and that the variations of volume be proportional to the original volume.

The difference $a - a'$ should fulfil the same conditions, and consequently also the difference $b' - b$, which is equal to it. But b' is the caloric necessary to raise the gas enclosed in *abcd* (Fig. 2) one degree; b' is the caloric surrendered by the gas when, enclosed in *abef*, it is cooled one degree. These quantities may serve as a measure for specific heats. We are then led to the establishment of the following proposition:

The change in the specific heat of a gas caused by change of volume depends entirely on the ratio between the original volume and the altered volume. That is, the difference of the specific heats does not depend on the absolute magnitude of the volumes, but only on their ratio.

This proposition might also be differently expressed, thus:

When a gas increases in volume in geometrical progression, its specific heat increases in arithmetical progression.

Thus, a being the specific heat of air taken at a given density, and $a + h$ the specific heat for a density one half less, it will be, for a density equal to one quarter, $a + 2h$; for a density equal to one eighth, $a + 3h$; and so on.

The specific heats are here taken with reference to weight. They are supposed to be taken at an

invariable volume, but, as we shall see, they would follow the same law if they were taken under constant pressure.

To what cause is the difference between specific heats at constant volume and at constant pressure really due? To the caloric required to produce in the second case increase of volume. Now, according to the law of Mariotte, increase of volume of a gas should be, for a given change of temperature, a determined fraction of the original volume, a fraction independent of pressure. According to the theorem expressed on page 76, if the ratio between the primitive volume and the altered volume is given, that determines the heat necessary to produce increase of volume. It depends solely on this ratio and on the weight of the gas. We must then conclude that:

The difference between specific heat at constant pressure and specific heat at constant volume is always the same, whatever may be the density of the gas, provided the weight remains the same.

These specific heats both increase accordingly as the density of the gas diminishes, but their difference does not vary.*

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* MM. Gay-Lussac and Welter have found by direct experiments, cited in the *Mécanique Céleste* and in the *Annales de Chimie et de Physique*, July, 1822, p. 267, that

Since the difference between the two capacities for heat is constant, if one increases in arithmetical progression the other should follow a similar progression: thus one law is applicable to specific heats at constant pressure.

We have tacitly assumed the increase of specific heat with that of volume. This increase is indicated by the experiments of MM. Delaroche and Bérard: in fact these physicists have found 0.967 for the specific heat of air under the pressure of

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the ratio between the specific heat at constant pressure and the specific heat at constant volume varies very little with the density of the gas. According to what we have just seen, the difference should remain constant, and not the ratio. As, further, the specific heat of gases for a given weight varies very little with the density, it is evident that the ratio itself experiences but slight changes.

The ratio between the specific heat of atmospheric air at constant pressure and at constant volume is, according to MM. Gay-Lussac and Welter, 1.3748, a number almost constant for all pressures, and even for all temperatures. We have come, through other considerations, to the number $\frac{267 + 116}{267} = 1.44$, which differs from the former $\frac{1}{20}$, and we have used this number to prepare a table of the specific heats of gases at constant volume. So we need not regard this table as very exact, any more than the table given on p. 89. These tables are mainly intended to demonstrate the laws governing specific heats of aeriform fluids.

1 metre of mercury (see Mémoire already cited), taking for the unit the specific heat of the same weight of air under the pressure of $0^{m}.760$.

According to the law that specific heats follow with relation to pressures, it is only necessary to have observed them in two particular cases to deduce them in all possible cases: it is thus that, making use of the experimental result of MM. Delaroche and Bérard which has just been given, we have prepared the following table of the specific heat of air under different pressures:

SPECIFIC HEAT OF AIR.

Pressure in Atmospheres.	Specific Heat, that of Air under Atmospheric Pressure being 1.	Pressure in Atmospheres.	Specific Heat, that of Air under Atmospheric Pressure being 1.
$\frac{1}{1024}$	1.840	1	1.000
$\frac{1}{512}$	1.756	2	0.916
$\frac{1}{256}$	1.672	4	0.832
$\frac{1}{128}$	1.588	8	0.748
$\frac{1}{64}$	1.504	16	0.664
$\frac{1}{32}$	1.420	32	0.580
$\frac{1}{16}$	1.336	64	0.496
$\frac{1}{8}$	1.252	128	0.412
$\frac{1}{4}$	1.165	256	0.328
$\frac{1}{2}$	1.084	512	0.244
1	1.000	1024	0.160

The first column is, as we see, a geometrical progression, and the second an arithmetical progression.

We have carried out the table to the extremes of compression and rarefaction. It may be believed that air would be liquefied before acquiring a density 1024 times its normal density, that is, before becoming more dense than water. The specific heat would become zero and even negative on extending the table beyond the last term. We think, furthermore, that the figures of the second column here decrease too rapidly. The experiments which serve as a basis for our calculation have been made within too contracted limits for us to expect great exactness in the figures which we have obtained, especially in the outside numbers.

Since we know, on the one hand, the law according to which heat is disengaged in the compression of gases, and on the other, the law according to which specific heat varies with volume, it will be easy for us to calculate the increase of temperature of a gas that has been compressed without being allowed to lose heat. In fact, the compression may be considered as composed of two successive operations: (1) compression at a constant temperature; (2) restoration of the caloric emitted. The temperature will rise through the second operation in inverse ratio with the specific heat acquired by the gas after the reduction of volume,—specific heat that we are able to calculate

by means of the law demonstrated above. The heat set free by compression, according to the theorem of page 81, ought to be represented by an expression of the form

$$s = A + B \log v,$$

s being this heat, v the volume of the gas after compression, A and B arbitrary constants dependent on the primitive volume of the gas, on its pressure, and on the units chosen.

The specific heat varying with the volume according to the law just demonstrated, should be represented by an expression of the form

$$z = A' + B' \log v,$$

A' and B' being the different arbitrary constants of A and B.

The increase of temperature acquired by the gas, as the effect of compression, is proportional to the ratio $\frac{s}{z}$ or to the relation $\frac{A + B \log v}{A' + B' \log v}$. It can be represented by this ratio itself; thus, calling it t, we shall have

$$t = \frac{A + B \log v}{A' + B' \log v}.$$

If the original volume of the gas is 1, and the original temperature zero, we shall have at the

same time $t = 0$, $\log v = 0$, whence $A = 0$; t will then express not only the increase of temperature, but the temperature itself above the thermometric zero.

We need not consider the formula that we have just given as applicable to very great changes in the volume of gases. We have regarded the elevation of temperature as being in inverse ratio to the specific heat; which tacitly supposes the specific heat to be constant at all temperatures. Great changes of volume lead to great changes of temperature in the gas, and nothing proves the constancy of specific heat at different temperatures, especially at temperatures widely separated. This constancy is only an hypothesis admitted for gases by analogy, to a certain extent verified for solid bodies and liquids throughout a part of the thermometric scale, but of which the experiments of MM. Dulong and Petit have shown the inaccuracy when it is desirable to extend it to temperatures far above 100°.*

According to a law of MM. Clement and Desormes, a law established by direct experiment, the vapor of water, under whatever pressure it may be formed, contains always, at equal weights, the

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* Note C, Appendix B.

same quantity of heat; which leads to the assertion that steam, compressed or expanded mechanically without loss of heat, will always be found in a saturated state if it was so produced in the first place. The vapor of water so made may then be regarded as a permanent gas, and should observe all the laws of one. Consequently the formula

$$t = \frac{A \ + B \ \log v}{A' + B' \log v}$$

should be applicable to it, and be found to accord with the table of tensions derived from the direct experiments of M. Dalton.

We may be assured, in fact, that our formula, with a convenient determination of arbitrary constants, represents very closely the results of experiment. The slight irregularities which we find therein do not exceed what we might reasonably attribute to errors of observation.*

We will return, however, to our principal subject, from which we have wandered too far—the motive power of heat.

We have shown that the quantity of motive power developed by the transfer of caloric from one body to another depends essentially upon the temperature of the two bodies, but we have not

* Note D, Appendix B.

shown the relation between these temperatures and the quantities of motive power produced. It would at first seem natural enough to suppose that for equal differences of temperature the quantities of motive power produced are equal; that is, for example, the passage of a given quantity of caloric from a body, A, maintained at 100°, to a body, B, maintained at 50°, should give rise to a quantity of motive power equal to that which would be developed by the transfer of the same caloric from a body, B, at 50°, to a body, C, at zero. Such a law would doubtless be very remarkable, but we do not see sufficient reason for admitting it *à priori*. We will investigate its reality by exact reasoning.

Let us imagine that the operations described on p. 70 be conducted successively on two quantities of atmospheric air equal in weight and volume, but taken at different temperatures. Let us suppose, further, the differences of temperature between the bodies A and B equal, so these bodies would have for example, in one of these cases, the temperatures 100° and $100° - h$ (h being indefinitely small), and in the other 1° and $1° - h$. The quantity of motive power produced is, in each case, the difference between that which the gas supplies by its dilatation and that which must be expended to restore its primitive volume. Now this differ-

ence is the same in both cases, as any one can prove by simple reasoning, which it seems unnecessary to give here in detail; hence the motive power produced is the same.

Let us now compare the quantities of heat employed in the two cases. In the first, the quantity of heat employed is that which the body *A* furnishes to the air to maintain it at the temperature of 100° during its expansion. In the second, it is the quantity of heat which this same body should furnish to it, to keep its temperature at one degree during an exactly similar change of volume. If these two quantities of heat were equal, there would evidently result the law that we have already assumed. But nothing proves that it is so, and we shall find that these quantities are not equal.

The air that we shall first consider as occupying the space *abcd* (Fig. 2), and having 1 degree of temperature, can be made to occupy the space *abef*, and to acquire the temperature of 100 degrees by two different means:

(1) We may heat it without changing its volume, then expand it, keeping its temperature constant.

(2) We may begin by expanding it, maintaining the temperature constant, then heat it, when it has acquired its greater volume.

Let a and b be the quantities of heat employed successively in the first of the two operations, and let b' and a' be the quantities of heat employed successively in the second. As the final result of these two operations is the same, the quantities of heat employed in both should be equal. We have then

$$a + b = a' + b',$$

whence

$$a' - a = b - b'.$$

a' is the quantity of heat required to cause the gas to rise from 1° to 100° when it occupies the space *abef*.

a is the quantity of heat required to cause the gas to rise from 1° to 100° when it occupies the space *abcd*.

The density of the air is less in the first than in the second case, and according to the experiments of MM. Delaroche and Bérard, already cited on page 87, its capacity for heat should be a little greater.

The quantity a' being found to be greater than the quantity a, b should be greater than b'. Consequently, generalizing the proposition, we should say:

The quantity of heat due to the change of volume of a gas is greater as the temperature is higher.

Thus, for example, more caloric is necessary to maintain at 100° the temperature of a certain quantity of air the volume of which is doubled, than to maintain at 1° the temperature of this same air during a dilatation exactly equal.

These unequal quantities of heat would produce, however, as we have seen, equal quantities of motive power for equal fall of caloric taken at different heights on the thermometric scale; whence we draw the following conclusion:

The fall of caloric produces more motive power at inferior than at superior temperatures.

Thus a given quantity of heat will develop more motive power in passing from a body kept at 1 degree to another maintained at zero, than if these two bodies were at the temperature of 101° and 100°.

The difference, however, should be very slight. It would be nothing if the capacity of the air for heat remained constant, in spite of changes of density. According to the experiments of MM. Delaroche and Bérard, this capacity varies little—so little even, that the differences noticed might strictly have been attributed to errors of observation or to some circumstances of which we have failed to take account.

We are not prepared to determine precisely,

with no more experimental data than we now possess, the law according to which the motive power of heat varies at different points on the thermometric scale. This law is intimately connected with that of the variations of the specific heat of gases at different temperatures—a law which experiment has not yet made known to us with sufficient exactness.*

We will endeavor now to estimate exactly the motive power of heat, and in order to verify our fundamental proposition, in order to determine whether the agent used to realize the motive power is really unimportant relatively to the quantity of this power, we will select several of them successively: atmospheric air, vapor of water, vapor of alcohol.

Let us suppose that we take first atmospheric air. The operation will proceed according to the method indicated on page 70. We will make the following hypotheses: The air is taken under atmospheric pressure. The temperature of the body A is $\frac{1}{1000}$ of a degree above zero, that of the body B is zero. The difference is, as we see, very slight—a necessary condition here.

The increase of volume given to the air in our

* Note E, Appendix B.

operation will be $\frac{1}{116} + \frac{1}{267}$ of the primitive volume; this is a very slight increase, absolutely speaking, but great relatively to the difference of temperature between the bodies *A* and *B*.

The motive power developed by the whole of the two operations described (page 70) will be very nearly proportional to the increase of volume and to the difference between the two pressures exercised by the air, when it is found at the temperatures 0°.001 and zero.

This difference is, according to the law of M. Gay-Lussac, $\frac{1}{267000}$ of the elastic force of the gas, or very nearly $\frac{1}{267000}$ of the atmospheric pressure.

The atmospheric pressure balances at 10.40 metres head of water; $\frac{1}{267000}$ of this pressure equals $\frac{1}{267000} \times 10^{m}.40$ of head of water.

As to the increase of volume, it is, by supposition, $\frac{1}{116} + \frac{1}{267}$ of the original volume, that is, of the volume occupied by one kilogram of air at zero, a volume equal to $0^{mc}.77$, allowing for the specific weight of the air. So then the product,

$$\left(\frac{1}{116} + \frac{1}{267}\right) \times 0.77 \times \frac{1}{267000} \times 10.40,$$

will express the motive power developed. This power is estimated here in cubic metres of water raised one metre.

If we carry out the indicated multiplications, we find the value of the product to be 0.000000372.

Let us endeavor now to estimate the quantity of heat employed to give this result; that is, the quantity of heat passed from the body *A* to the body *B*.

The body *A* furnishes:

(1) The heat required to carry the temperature of one kilogram of air from zero to 0°.001;

(2) The quantity necessary to maintain at this temperature the temperature of the air when it experiences a dilatation of

$$\tfrac{1}{116} + \tfrac{1}{267}.$$

The first of these quantities of heat being very small in comparison with the second, we may disregard it. The second is, according to the reasoning on page 74, equal to that which would be necessary to increase one degree the temperature of one kilogram of air subjected to atmospheric pressure.

According to the experiments of MM. Delaroche and Bérard on the specific heat of gases, that of air is, for equal weights, 0.267 that of water. If, then, we take for the unit of heat the quantity necessary to raise 1 kilogram of water 1 degree,

that which will be required to raise 1 kilogram of air 1 degree would have for its value 0.267. Thus the quantity of heat furnished by the body A is

0.267 units.

This is the heat capable of producing 0.000000372 units of motive power by its fall from 0°.001 to zero.

For a fall a thousand times greater, for a fall of one degree, the motive power will be very nearly a thousand times the former, or

0.000372.

If, now, instead of 0.267 units of heat we employ 1000 units, the motive power produced will be expressed by the proportion

$$\frac{0.267}{0.000372} = \frac{1000}{x}, \quad \text{whence} \quad x = \frac{372}{267} = 1.395.$$

Thus 1000 units of heat passing from a body maintained at the temperature of 1 degree to another body maintained at zero would produce, in acting upon the air,

1.395 units of motive power.

We will now compare this result with that furnished by the action of heat on the vapor of water.

Let us suppose one kilogram of liquid water enclosed in the cylindrical vessel *abcd* (Fig. 4), between the bottom *ab* and the piston *cd*. Let us suppose, also, the two bodies *A*, *B* maintained each at a constant temperature, that of *A* being a very little above that of *B*. Let us imagine now the following operations:

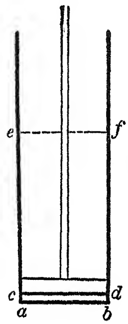

FIG. 4.

(1) Contact of the water with the body *A*, movement of the piston from the position *cd* to the position *ef*, formation of steam at the temperature of the body *A* to fill the vacuum produced by the extension of volume. We will suppose the space *abef* large enough to contain all the water in a state of vapor.

(2) Removal of the body *A*, contact of the vapor with the body *B*, precipitation of a part of this vapor, diminution of its elastic force, return of the piston from *ef* to *ab*, liquefaction of the rest of the vapor through the effect of the pressure combined with the contact of the body *B*.

(3) Removal of the body *B*, fresh contact of the water with the body *A*, return of the water to the temperature of this body, renewal of the former period, and so on.

The quantity of motive power developed in a

complete cycle of operations is measured by the product of the volume of the vapor multiplied by the difference between the tensions that it possesses at the temperature of the body A and at that of the body B. As to the heat employed, that is to say, transported from the body A to the body B, it is evidently that which was necessary to turn the water into vapor, disregarding always the small quantity required to restore the temperature of the liquid water from that of B to that of A.

Suppose the temperature of the body A 100 degrees, and that of the body B 99 degrees: the difference of the tensions will be, according to the table of M. Dalton, 26 millimetres of mercury or $0^{m}.36$ head of water.

The volume of the vapor is 1700 times that of the water. If we operate on one kilogram, that will be 1700 litres, or $1^{mc}.700$.

Thus the value of the motive power developed is the product

$$1.700 \times 0.36 = 0.611 \text{ units,}$$

of the kind of which we have previously made use.

The quantity of heat employed is the quantity required to turn into vapor water already heated to 100°. This quantity is found by experiment. We

have found it equal to 550°, or, to speak more exactly, to 550 of our units of heat.

Thus 0.611 units of motive power result from the employment of 550 units of heat. The quantity of motive power resulting from 1000 units of heat will be given by the proportion

$$\frac{550}{0.611} = \frac{1000}{x}, \quad \text{whence} \quad x = \frac{611}{550} = 1.112.$$

Thus 1000 units of heat transported from one body kept at 100 degrees to another kept at 99 degrees will produce, acting upon vapor of water, 1.112 units of motive power.

The number 1.112 differs by about $\frac{1}{4}$ from the number 1.395 previously found for the value of the motive power developed by 1000 units of heat acting upon the air; but it should be observed that in this case the temperatures of the bodies A and B were 1 degree and zero, while here they are 100 degrees and 99 degrees. The difference is much the same; but it is not found at the same height in the thermometric scale. To make an exact comparison, it would have been necessary to estimate the motive power developed by the steam formed at 1 degree and condensed at zero. It would also have been necessary to know the quantity of heat contained in the steam formed at one degree.

The law of MM. Clement and Desormes referred to on page 92 gives us this datum. The constituent heat of vapor of water being always the same at any temperature at which vaporization takes place, if 550 degrees of heat are required to vaporize water already brought up to 100 degrees, 550 + 100 or 650 will be required to vaporize the same weight of water taken at zero.

Making use of this datum and reasoning exactly as we did for water at 100 degrees, we find, as is easily seen,

$$1.290$$

for the motive power developed by 1000 units of heat acting upon the vapor of water between one degree and zero. This number approximates more closely than the first to

$$1.395.$$

It differs from it only $\frac{1}{13}$, an error which does not exceed probable limits, considering the great number of data of different sorts of which we have been obliged to make use in order to arrive at this approximation. Thus is our fundamental law verified in a special case.*

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* We find (*Annales de Chimie et de Physique*, July, 1818, p. 294) in a memoir of M. Petit an estimate of the motive power of heat applied to air and to vapor of water. This

We will examine another case in which vapor of alcohol is acted upon by heat. The reasoning is precisely the same as for the vapor of water. The data alone are changed. Pure alcohol boils under ordinary pressure at 78°.7 Centigrade. One kilogram absorbs, according to MM. Delaroche and Bérard, 207 units of heat in undergoing transformation into vapor at this same temperature, 78°.7.

The tension of the vapor of alcohol at one degree below the boiling-point is found to be diminished $\frac{1}{25}$. It is $\frac{1}{25}$ less than the atmospheric pressure; at least, this is the result of the experiment of M. Bétancour reported in the second part of *l'Architecture hydraulique* of M. Prony, pp. 180, 195.*

If we use these data, we find that, in acting upon one kilogram of alcohol at the temperatures of 78°.7 and 77°.7, the motive power developed will be 0.251 units.

This results from the employment of 207 units of heat. For 1000 units the proportion must be

$$\frac{207}{0.254} = \frac{1000}{x}, \text{ whence } x = 1.230.$$

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estimate leads us to attribute a great advantage to atmospheric air, but it is derived by a method of considering the action of heat which is quite imperfect.

* Note F, Appendix B.

This number is a little more than the 1.112 resulting from the use of the vapor of water at the temperatures 100° and 99°; but if we suppose the vapor of water used at the temperatures 78° and 77°, we find, according to the law of MM. Clement and Desorme, 1.212 for the motive power due to 1000 units of heat. This latter number approaches, as we see, very nearly to 1.230. There is a difference of only $\frac{1}{50}$.

We should have liked to be able to make other approximations of this sort—to be able to calculate, for example, the motive power developed by the action of heat on solids and liquids, by the congelation of water, and so on; but Physics as yet refuses us the necessary data.*

The fundamental law that we propose to confirm seems to us to require, however, in order to be placed beyond doubt, new verifications. It is based upon the theory of heat as it is understood to-day, and it should be said that this foundation does not appear to be of unquestionable solidity. New experiments alone can decide the question. Meanwhile we can apply the theoretical ideas expressed

* Those that we need are the expansive force acquired by solids and liquids by a given increase of temperature, and the quantity of heat absorbed or relinquished in the changes of volume of these bodies.

above, regarding them as exact, to the examination of the different methods proposed up to date, for the realization of the motive power of heat.

It has sometimes been proposed to develop motive power by the action of heat on solid bodies. The mode of procedure which naturally first occurs to the mind is to fasten immovably a solid body—a metallic bar, for example—by one of its extremities; to attach the other extremity to a movable part of the machine; then, by successive heating and cooling, to cause the length of the bar to vary, and so to produce motion. Let us try to decide whether this method of developing motive power can be advantageous. We have shown that the condition of the most effective employment of heat in the production of motion is, that all changes of temperature occurring in the bodies should be due to changes of volume. The nearer we come to fulfilling this condition the more fully will the heat be utilized. Now, working in the manner just described, we are very far from fulfilling this condition: change of temperature is not due here to change of volume; all the changes are due to contact of bodies differently heated—to the contact of the metallic bar, either with the body charged with furnishing heat to it, or with the body charged with carrying it off.

The only means of fulfilling the prescribed condition would be to act upon the solid body exactly as we did on the air in the operations described on page 92. But for this we must be able to produce, by a single change of volume of the solid body, considerable changes of temperature, that is, if we should want to utilize considerable falls of caloric. Now this appears impracticable. In short, many considerations lead to the conclusion that the changes produced in the temperature of solid or liquid bodies through the effect of compression and rarefaction would be but slight.

(1) We often observe in machines (particularly in steam-engines) solid pieces which endure considerable strain in one way or another, and although these efforts may be sometimes as great as the nature of the substances employed permits, the variations of temperature are scarcely perceptible.

(2) In the action of striking medals, in that of the rolling-mill, of the draw-plate, the metals undergo the greatest compression to which we can submit them, employing the hardest and strongest tools. Nevertheless the elevation of temperature is not great. If it were, the pieces of steel used in these operations would soon lose their temper.

(3) We know that it would be necessary to exert

on solids and liquids a very great strain in order to produce in them a reduction of volume comparable to that which they experience in cooling (cooling from 100° to zero, for example). Now the cooling requires a greater abstraction of caloric than would simple reduction of volume. If this reduction were produced by mechanical means, the heat set free would not then be able to make the temperature of the body vary as many degrees as the cooling makes it vary. It would, however, necessitate the employment of a force undoubtedly very considerable.

Since solid bodies are susceptible of little change of temperature through changes of volume, and since the condition of the most effective employment of heat for the development of motive power is precisely that all change of temperature should be due to a change of volume, solid bodies appear but ill fitted to realize this power.

The same remarks apply to liquids. The same reasons may be given for rejecting them.*

We are not speaking now of practical difficulties.

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* The recent experiments of M. Oerstedt on the compressibility of water have shown that, for a pressure of five atmospheres, the temperature of this liquid exhibits no appreciable change. (See *Annales de Chimie et de Physique*, Feb. 1823, p. 192.)

They will be numberless. The motion produced by the dilatation and compression of solid or liquid bodies would only be very slight. In order to give them sufficient amplitude we should be forced to make use of complicated mechanisms. It would be necessary to employ materials of the greatest strength to transmit enormous pressure; finally, the successive operations would be executed very slowly compared to those of the ordinary steam-engine, so that apparatus of large dimensions and heavy cost would produce but very ordinary results.

The elastic fluids, gases or vapors, are the means really adapted to the development of the motive power of heat. They combine all the conditions necessary to fulfil this office. They are easy to compress; they can be almost infinitely expanded; variations of volume occasion in them great changes of temperature; and, lastly, they are very mobile, easy to heat and to cool, easy to transport from one place to another, which enables them to produce rapidly the desired effects. We can easily conceive a multitude of machines fitted to develop the motive power of heat through the use of elastic fluids; but in whatever way we look at it, we should not lose sight of the following principles:

(1) The temperature of the fluid should be made as high as possible, in order to obtain a great fall of caloric, and consequently a large production of motive power.

(2) For the same reason the cooling should be carried as far as possible.

(3) It should be so arranged that the passage of the elastic fluid from the highest to the lowest temperature should be due to increase of volume; that is, it should be so arranged that the cooling of the gas should occur spontaneously as the effect of rarefaction. The limits of the temperature to which it is possible to bring the fluid primarily, are simply the limits of the temperature obtainable by combustion; they are very high.

The limits of cooling are found in the temperature of the coldest body of which we can easily and freely make use; this body is usually the water of the locality.

As to the third condition, it involves difficulties in the realization of the motive power of heat when the attempt is made to take advantage of great differences of temperature, to utilize great falls of heat. In short, it is necessary then that the gas, by reason of its rarefaction, should pass from a very high temperature to a very low one, which requires a great change of volume and of

density, which requires also that the gas be first taken under a very heavy pressure, or that it acquire by its dilatation an enormous volume—conditions both difficult to fulfil. The first necessitates the employment of very strong vessels to contain the gas at a very high temperature and under very heavy pressure. The second necessitates the use of vessels of large dimensions. These are, in a word, the principal obstacles which prevent the utilization in steam-engines of a great part of the motive power of the heat. We are obliged to limit ourselves to the use of a slight fall of caloric, while the combustion of the coal furnishes the means of procuring a very great one.

It is seldom that in steam-engines the elastic fluid is produced under a higher pressure than six atmospheres—a pressure corresponding to about 160° Centigrade, and it is seldom that condensation takes place at a temperature much under 40°. The fall of caloric from 160° to 40° is 120°, while by combustion we can procure a fall of 1000° to 2000°.

In order to comprehend this more clearly, let us recall what we have termed the fall of caloric. This is the passage of the heat from one body, *A*, having an elevated temperature, to another, *B*, where it is lower. We say that the fall of the

caloric is 100° or 1000° when the difference of temperature between the bodies *A* and *B* is 100° or 1000°.

In a steam-engine which works under a pressure of six atmospheres the temperature of the boiler is 160°. This is the body *A*. It is kept, by contact with the furnace, at the constant temperature of 160°, and continually furnishes the heat necessary for the formation of steam. The condenser is the body *B*. By means of a current of cold water it is kept at a nearly constant temperature of 40°. It absorbs continually the caloric brought from the body *A* by the steam. The difference of temperature between these two bodies is 160° — 40°, or 120°. Hence we say that the fall of caloric is here 120°.

Coal being capable of producing, by its combustion, a temperature higher than 1000°, and the cold water, which is generally used in our climate, being at about 10°, we can easily procure a fall of caloric of 1000°, and of this only 120° are utilized by steam-engines. Even these 120° are not wholly utilized. There is always considerable loss due to useless re-establishments of equilibrium in the caloric.

It is easy to see the advantages possessed by high-pressure machines over those of lower pressure. *This superiority lies essentially in the power*

of utilizing a greater fall of caloric. The steam produced under a higher pressure is found also at a higher temperature, and as, further, the temperature of condensation remains always about the same, it is evident that the fall of caloric is more considerable. But to obtain from high-pressure engines really advantageous results, it is necessary that the fall of caloric should be most profitably utilized. It is not enough that the steam be produced at a high temperature: it is also necessary that by the expansion of its volume its temperature should become sufficiently low. A good steam-engine, therefore, should not only employ steam under heavy pressure, but *under successive and very variable pressures, differing greatly from one another, and progressively decreasing.**

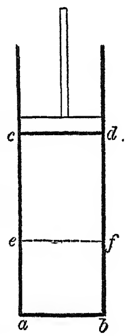

Fig. 5.

In order to understand in some sort *à posteriori* the advantages of high-pressure engines, let us suppose steam to be formed under atmospheric pressure and introduced into the cylindrical vessel *abcd* (Fig. 5), under the piston *cd*, which at first touches the bottom *ab*. The steam, after having moved the piston from *ab* to *cd*, will continue

* Note G, Appendix B.

finally to produce its results in a manner with which we will not concern ourselves.

Let us suppose that the piston having moved to *cd* is forced downward to *ef*, without the steam being allowed to escape, or any portion of its caloric to be lost. It will be driven back into the space *abef*, and will increase at the same time in density, elastic force, and temperature. If the steam, instead of being produced under atmospheric pressure, had been produced just when it was being forced back into *abef*, and so that after its introduction into the cylinder it had made the piston move from *ab* to *ef*, and had moved it simply by its extension of volume, from *ef* to *cd*, the motive power produced would have been more considerable than in the first case. In fact, the movement of the piston, while equal in extent, would have taken place under the action of a greater pressure, though variable, and though progressively decreasing.

The steam, however, would have required for its formation exactly the same quantity of caloric, only the caloric would have been employed at a higher temperature.

It is considerations of this nature which have led to the making of double-cylinder engines—engines invented by Mr. Hornblower, improved by Mr. Woolf, and which, as regards economy of the com-

bustible, are considered the best. They consist of a small cylinder, which at each pulsation is filled more or less (often entirely) with steam, and of a second cylinder having usually a capacity quadruple that of the first, and which receives no steam except that which has already operated in the first cylinder. Thus the steam when it ceases to act has at least quadrupled in volume. From the second cylinder it is carried directly into the condenser, but it is conceivable that it might be carried into a third cylinder quadruple the second, and in which its volume would have become sixteen times the original volume. The principal obstacle to the use of a third cylinder of this sort is the capacity which it would be necessary to give it, and the large dimensions which the openings for the passage of the steam must have. We will say no more on this subject, as we do not propose here to enter into the details of construction of steam-engines. These details call for a work devoted specially to them, and which does not yet exist, at least in France.*

* We find in the work called *De la Richesse Minérale*, by M. Heron de Villefosse, vol. iii. p. 50 and following, a good description of the steam-engines actually in use in mining. In England the steam-engine has been very fully discussed in the *Encyclopedia Britannica*. Some of the data here employed are drawn from the latter work.

If the expansion of the steam is mainly limited by the dimensions of the vessels in which the dilatation must take place, the degree of condensation at which it is possible to use it at first is limited only by the resistance of the vessels in which it is produced, that is, of the boilers.

In this respect we have by no means attained the best possible results. The arrangement of the boilers generally in use is entirely faulty, although the tension of the steam rarely exceeds from four to six atmospheres. They often burst and cause severe accidents. It will undoubtedly be possible to avoid such accidents, and meantime to raise the steam to much greater pressures than is usually done.

Besides the high-pressure double-cylinder engines of which we have spoken, there are also high-pressure engines of one cylinder. The greater part of these latter have been constructed by two ingenious English engineers, Messrs. Trevithick and Vivian. They employ the steam under a very high pressure, sometimes eight to ten atmospheres, but they have no condenser. The steam, after it has been introduced into the cylinder, undergoes therein a certain increase of volume, but preserves always a pressure higher than atmospheric. When it has fulfilled its office it is thrown out into the

atmosphere. It is evident that this mode of working is fully equivalent, in respect to the motive power produced, to condensing the steam at 100°, and that a portion of the useful effect is lost. But the engines working thus dispense with condenser and air-pump. They are less costly than the others, less complicated, occupy less space, and can be used in places where there is not sufficient water for condensation. In such places they are of inestimable advantage, since no others could take their place. These engines are principally employed in England to move coal-wagons on railroads laid either in the interior of mines or outside of them.

We have, further, only a few remarks to make upon the use of permanent gases and other vapors than that of water in the development of the motive power of heat.

Various attempts have been made to produce motive power by the action of heat on atmospheric air. This gas presents, as compared with vapor of water, both advantages and disadvantages, which we will proceed to examine.

(1) It presents, as compared with vapor of water, a notable advantage in that, having for equal volume a much less capacity for heat, it would cool more rapidly by an equal increase of volume.

(This fact is proved by what has already been stated.) Now we have seen how important it is to produce by change of volume the greatest possible changes of temperature.

(2) Vapors of water can be formed only through the intervention of a boiler, while atmospheric air could be heated directly by combustion carried on within its own mass. Considerable loss could thus be prevented, not only in the quantity of heat, but also in its temperature. This advantage belongs exclusively to atmospheric air. Other gases do not possess it. They would be even more difficult to heat than vapor of water.

(3) In order to give to air great increase of volume, and by that expansion to produce a great change of temperature, it must first be taken under a sufficiently high pressure; then it must be compressed with a pump or by some other means before heating it. This operation would require a special apparatus, an apparatus not found in steam-engines. In the latter, water is in a liquid state when injected into the boiler, and to introduce it requires but a small pump.

(4) The condensing of the vapor by contact with the refrigerant body is much more prompt and much easier than is the cooling of air. There might, of course, be the expedient of throwing the

latter out into the atmosphere, and there would be also the advantage of avoiding the use of a refrigerant, which is not always available, but it would be requisite that the increase of the volume of the air should not reduce its pressure below that of the atmosphere.

(5) One of the gravest inconveniences of steam is that it cannot be used at high temperatures without necessitating the use of vessels of extraordinary strength. It is not so with air for which there exists no necessary relation between the elastic force and the temperature. Air, then, would seem more suitable than steam to realize the motive power of falls of caloric from high temperatures. Perhaps in low temperatures steam may be more convenient. We might conceive even the possibility of making the same heat act successively upon air and vapor of water. It would be only necessary that the air should have, after its use, an elevated temperature, and instead of throwing it out immediately into the atmosphere, to make it envelop a steam-boiler, as if it issued directly from a furnace.

The use of atmospheric air for the development of the motive power of heat presents in practice very great, but perhaps not insurmountable, difficulties. If we should succeed in overcoming them,

it would doubtless offer a notable advantage over vapor of water.*

As to the other permanent gases, they should be absolutely rejected. They have all the inconveniences of atmospheric air, with none of its advantages. The same can be said of other vapors than that of water, as compared with the latter.

If we could find an abundant liquid body which would vaporize at a higher temperature than water, of which the vapor would have, for the same volume, a less specific heat, which would not attack the metals employed in the construction of machines, it would undoubtedly merit the preference. But nature provides no such body.

The use of the vapor of alcohol has been proposed. Machines have even been constructed for the purpose of using it, by avoiding the mixture of its vapor with the water of condensation, that is, by applying the cold body externally instead of introducing it into the machine. It has been thought that a remarkable advantage might be secured by using the vapor of alcohol in that it possesses a stronger tension than the vapor of water at the same temperature. We can see in this only a fresh obstacle to be overcome. The principal defect of

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* Note I, Appendix B.

the vapor of water is its excessive tension at an elevated temperature; now this defect exists still more strongly in the vapor of alcohol. As to the relative advantage in a greater production of motive power,—an advantage attributed to it,—we know by the principles above demonstrated that it is imaginary.

It is thus upon the use of atmospheric air and vapor of water that subsequent attempts to perfect heat-engines should be based. It is to utilize by means of these agents the greatest possible falls of caloric that all efforts should be directed.

Finally, we will show how far we are from having realized, by any means at present known, all the motive power of combustibles.

One kilogram of carbon burnt in the calorimeter furnishes a quantity of heat capable of raising one degree Centigrade about 7000 kilograms of water, that is, it furnishes 7000 units of heat according to the definition of these units given on page 100.

The greatest fall of caloric attainable is measured by the difference between the temperature produced by combustion and that of the refrigerant bodies. It is difficult to perceive any other limits to the temperature of combustion than those in which the combination between oxygen and the combustible may take place. Let us assume, how-

ever, that 1000° may be this limit, and we shall certainly be below the truth. As to the temperature of the refrigerant, let us suppose it 0°. We estimated approximately (page 104) the quantity of motive power that 1000 units of heat develop between 100° and 99°. We found it to be 1.112 units of power, each equal to 1 metre of water raised to a height of 1 metre.

If the motive power were proportional to the fall of caloric, if it were the same for each thermometric degree, nothing would be easier than to estimate it from 1000° to 0°. Its value would be

$$1.112 \times 1000 = 1112.$$

But as this law is only approximate, and as possibly it deviates much from the truth at high temperatures, we can only make a very rough estimate. We will suppose the number 1112 reduced one-half, that is, to 560.

Since a kilogram of carbon produces 7000 units of heat, and since the number 560 is relatively 1000 units, it must be multiplied by 7, which gives

$$7 \times 560 = 3920.$$

This is the motive power of 1 kilogram of carbon. In order to compare this theoretical result with

that of experiment, let us ascertain how much motive power a kilogram of carbon actually develops in the best-known steam-engines.

The engines which, up to this time, have shown the best results are the large double-cylinder engines used in the drainage of the tin and copper mines of Cornwall. The best results that have been obtained with them are as follows:

65 millions of lbs. of water have been raised one English foot by the bushel of coal burned (the bushel weighing 88 lbs.). This is equivalent to raising, by a kilogram of coal, 195 cubic metres of water to a height of 1 metre, producing thereby 195 units of motive power per kilogram of coal burned.

195 units are only the twentieth of 3920, the theoretical maximum; consequently $\frac{1}{20}$ only of the motive power of the combustible has been utilized.

We have, nevertheless, selected our example from among the best steam-engines known.

Most engines are greatly inferior to these. The old engine of Chaillot, for example, raised twenty cubic metres of water thirty-three metres, for thirty kilograms of coal consumed, which amounts to twenty-two units of motive power per kilogram, —a result nine times less than that given above,

and one hundred and eighty times less than the theoretical maximum.

We should not expect ever to utilize in practice all the motive power of combustibles. The attempts made to attain this result would be far more hurtful than useful if they caused other important considerations to be neglected. The economy of the combustible is only one of the conditions to be fulfilled in heat-engines. In many cases it is only secondary. It should often give precedence to safety, to strength, to the durability of the engine, to the small space which it must occupy, to small cost of installation, etc. To know how to appreciate in each case, at their true value, the considerations of convenience and economy which may present themselves; to know how to discern the more important of those which are only accessories; to balance them properly against each other, in order to attain the best results by the simplest means: such should be the leading characteristics of the man called to direct, to co-ordinate among themselves the labors of his comrades, to make them co-operate towards one useful end, of whatsoever sort it may be.

IV. *

CARNOT'S THEORY OF THE MOTIVE POWER OF HEAT. †

WITH NUMERICAL RESULTS DEDUCED FROM REGNAULT'S EXPERIMENTS ON STEAM. ‡

By Sir William Thomson.

1. The presence of heat may be recognized in every natural object; and there is scarcely an operation in nature which is not more or less

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* From *Transactions of the Edinburgh Royal Society*, xiv. 1849; *Annales de Chimie*, xxxv. 1852.

* Published in 1824, in a work entitled "*Réflexions sur la Puissance Motrice du Feu, et sur les Machines Propres à Developer cette Puissance. Par S. Carnot.*" [Note of Nov. 5, 1881. The original work has now been republished, with a biographical notice, Paris, 1878.]

‡ An account of the first part of a series of researches undertaken by Mons. Regnault, by order of the late French Government, for ascertaining the various physical data of importance in the theory of the steam-engine, has

affected by its all-pervading influence. An evolution and subsequent absorption of heat generally give rise to a variety of effects; among which may be enumerated, chemical combinations or decompositions; the fusion of solid substances; the vaporization of solids or liquids; alterations in the dimensions of bodies, or in the statical pressure by which their dimensions may be modified; mechanical resistance overcome; electrical currents generated. In many of the actual phenomena of nature several or all of these effects are produced together; and their complication will, if we attempt to trace the agency of heat in producing any individual effect, give rise to much perplexity. It will, therefore, be desirable, in laying the foundation of a physical theory of any of the effects of heat, to discover or to imagine phenomena free from all such complication, and depending on a definite thermal agency; in which the relation between the cause and effect, traced

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been recently published (under the title "*Relation des Expériences,*" etc.) in the *Mémoires de l'Institut,* of which it constitutes the twenty-first volume (1847). The second part of these researches has not yet been published. [Note of Nov. 5, 1881. The continuation of these researches has now been published; thus we have for the whole series, vol. i. in 1847; vol. ii. in 1862; and vol. iii. in 1870.]

through the medium of certain simple operations, may be clearly appreciated. Thus it is that Carnot, in accordance with the strictest principles of philosophy, enters upon the investigation of the theory of the motive power of heat.

2. The sole effect to be contemplated in investigating the motive power of heat is *resistance overcome*, or, as it is frequently called, "*work performed*," or "*mechanical effect*." The questions to be resolved by a complete theory of the subject are the following:

(1) What is the precise nature of the thermal agency by means of which *mechanical effect* is to be produced, without effects of any other kind?

(2) How may the amount of this thermal agency necessary for performing a given quantity of work be estimated?

3. In the following paper I shall commence by giving a short abstract of the reasoning by which Carnot is led to an answer to the first of these questions; I shall then explain the investigation by which, in accordance with his theory, the experimental elements necessary for answering the second question are indicated; and, in conclusion, I shall state the *data* supplied by Regnault's recent observations on steam, and apply them to obtain, as approximately as the present state of experi-

mental science enables us to do, a complete solution of the question.

I. On the nature of Thermal agency, considered as a motive power.

4. There are [at present known] two, and only two, distinct ways in which mechanical effect can be obtained from heat. One of these is by means of the alterations of volume, which bodies may experience through the action of heat; the other is through the medium of electric agency. Seebeck's discovery of thermo-electric currents enables us at present to conceive of an electro-magnetic engine supplied from a thermal origin, being used as a motive power; but this discovery was not made until 1821, and the subject of thermo-electricity can only have been generally known in a few isolated facts, with reference to the electrical effects of heat upon certain crystals, at the time when Carnot wrote. He makes no allusion to it, but confines himself to the method for rendering thermal agency available as a source of mechanical effect, by means of the expansions and contractions of bodies.

5. A body expanding or contracting under the action of force may, in general, either produce mechanical effect by overcoming resistance, or receive mechanical effect by yielding to the action

of force. The amount of mechanical effect thus developed will depend not only on the calorific agency concerned, but also on the alteration in the physical condition of the body. Hence, after allowing the volume and temperature of the body to change, we must restore it to its original temperature and volume; and then we may estimate the aggregate amount of mechanical effect developed as due solely to the thermal origin.

6. Now the ordinarily-received, and almost universally-acknowledged, principles with reference to "quantities of caloric" and "latent heat" lead us to conceive that, at the end of a cycle of operations, when a body is left in precisely its primitive physical condition, if it has absorbed any heat during one part of the operations, it must have given out again exactly the same amount during the remainder of the cycle. The truth of this principle is considered as axiomatic by Carnot, who admits it as the foundation of his theory; and expresses himself in the following terms regarding it, in a note on one of the passages of his treatise:*

"In our demonstrations we tacitly assume that after a body has experienced a certain number of transformations, if it be brought identically to its

* Carnot, p. 67.

primitive physical state as to density, temperature, and molecular constitution, it must contain the same quantity of heat as that which it initially possessed; or, in other words, we suppose that the quantities of heat lost by the body under one set of operations are precisely compensated by those which are absorbed in the others. This fact has never been doubted; it has at first been admitted without reflection, and afterwards verified, in many cases, by calorimetrical experiments. To deny it would be to overturn the whole theory of heat, in which it is the fundamental principle. It must be admitted, however, that the chief foundations on which the theory of heat rests, would require a most attentive examination. Several experimental facts appear nearly inexplicable in the actual state of this theory."

7. Since the time when Carnot thus expressed himself, the necessity of a most careful examination of the entire experimental basis of the theory of heat has become more and more urgent. Especially all those assumptions depending on the idea that heat is a *substance*, invariable in quantity; not convertible into any other element, and incapable of being *generated* by any physical agency; in fact the acknowledged principles of latent heat,—would require to be tested by a most

searching investigation before they ought to be admitted, as they usually have been, by almost every one who has been engaged on the subject, whether in combining the results of experimental research, or in general theoretical investigations.

8. The extremely important discoveries recently made by Mr. Joule of Manchester, that heat is evolved in every part of a closed electric conductor, moving in the neighborhood of a magnet,* and

* The *evolution* of heat in a fixed conductor, through which a galvanic current is sent from any source whatever, has long been known to the scientific world; but it was pointed out by Mr. Joule that we cannot infer from any previously-published experimental researches, the actual *generation* of heat when the current originates in electro-magnetic induction; since the question occurs, *is the heat which is evolved in one part of the closed conductor merely transferred from those parts which are subject to the inducing influence?* Mr. Joule, after a most careful experimental investigation with reference to this question, finds that it must be answered in the negative. (See a paper "On the Calorific Effects of Magneto-Electricity, and on the Mechanical Value of Heat; by J. P. Joule, Esq." Read before the British Association at Cork in 1843, and subsequently communicated by the Author to the *Philosophical Magazine*, vol. xxiii., pp. 263, 347, 435.)

Before we can finally conclude that heat is absolutely generated in such operations, it would be necessary to prove that the inducing magnet does not become lower in

that heat is *generated* by the friction of fluids in motion, seem to overturn the opinion commonly held that heat cannot be *generated*, but only produced from a source, where it has previously existed either in a sensible or in a latent condition.

In the present state of science, however, no operation is known by which heat can be absorbed into a body without either elevating its temperature or becoming latent, and producing some alteration in its physical condition; and the fundamental axiom adopted by Carnot may be considered as still the most probable basis for an investigation of the motive power of heat; although this, and with it every other branch of the theory of heat, may ultimately require to be reconstructed upon another foundation, when our experimental data are more complete. On this understanding, and to avoid a

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temperature, and thus compensate for the heat evolved in the conductor. I am not aware that any examination with reference to the truth of this conjecture has been instituted; but, in the case where the inducing body is a pure electro-magnet (without any iron), the experiments actually performed by Mr. Joule render the conclusion probable that the heat evolved in the wire of the electro-magnet is not affected by the inductive action, otherwise than through the reflected influence which increases the strength of its own current.

repetition of doubts, I shall refer to Carnot's fundamental principle, in all that follows, as if its truth were thoroughly established.

9. We are now led to the conclusion that the origin of motive power, developed by the alternate expansions and contractions of a body, must be found in the agency of heat entering the body and leaving it; since there cannot, at the end of a complete cycle, when the body is restored to its primitive physical condition, have been any absolute absorption of heat, and consequently no conversion of heat, or caloric, into mechanical effect; and it remains for us to trace the precise nature of the circumstances under which heat must enter the body, and afterwards leave it, so that mechanical effect may be produced. As an example, we may consider that machine for obtaining motive power from heat with which we are most familiar—the steam-engine.

10. Here, we observe, that heat enters the machine from the furnace, through the sides of the boiler, and that heat is continually abstracted by the water employed for keeping the condenser cool. According to Carnot's fundamental principle, the quantity of heat thus discharged, during a complete revolution (or double stroke) of the engine, must be precisely equal to that which enters the water of

the boiler;* provided the total mass of water and steam be invariable, and be restored to its primitive physical condition (which will be the case rigorously, if the condenser be kept cool by the external application of cold water instead of by injection, as is more usual in practice), and if the condensed water be restored to the boiler at the end of each complete revolution. Thus we perceive that a certain quantity of heat is *let down* from a hot body, the metal of the boiler, to another body at a lower temperature, the metal of the condenser; and that there results from this transference of heat a certain development of mechanical effect.

11. If we examine any other case in which mechanical effect is obtained from a thermal origin, by means of the alternate expansions and contractions of any substance whatever, instead of the water of a steam-engine, we find that a similar transference of heat is effected, and we may therefore answer the first question proposed, in the following manner:

The thermal agency by which mechanical effect may be obtained is the transference of heat from one body to another at a lower temperature.

* So generally is Carnot's principle tacitly admitted as an axiom, that its application in this case has never, so far as I am aware, been questioned by practical engineers. (1849).

II. On the measurement of Thermal Agency, considered with reference to its equivalent of mechanical effect.

12. A *perfect* thermodynamic engine of any kind is a machine by means of which the greatest possible amount of mechanical effect can be obtained from a given thermal agency; and, therefore, if in any manner we can construct or imagine a perfect engine which may be applied for the transference of a given quantity of heat from a body at any given temperature to another body at a lower given temperature, and if we can evaluate the mechanical effect thus obtained, we shall be able to answer the question at present under consideration, and so to complete the theory of the motive power of heat. But whatever kind of engine we may consider with this view, it will be necessary for us to prove that it is a perfect engine; since the transference of the heat from one body to the other may be wholly, or partially, effected by conduction through a solid,* without the development of

* When "thermal agency" is thus spent in conducting heat through a solid, what becomes of the mechanical effect which it might produce? Nothing can be lost in the operations of nature—no energy can be destroyed. What effect, then, is produced in place of the mechanical effect which is lost? A perfect theory of heat impera-

mechanical effect; and, consequently, engines may be constructed in which the whole or any portion

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tively demands an answer to this question; yet no answer can be given in the present state of science. A few years ago, a similar confession must have been made with reference to the mechanical effect lost in a fluid set in motion in the interior of a rigid closed vessel, and allowed to come to rest by its own internal friction; but in this case the foundation of a solution of the difficulty has been actually found in Mr. Joule's discovery of the generation of heat, by the internal friction of a fluid in motion. Encouraged by this example, we may hope that the very perplexing question in the theory of heat, by which we are at present arrested, will before long be cleared up. [Note of Sept., 1881. The Theory of the Dissipation of Energy completely answers this question and removes the difficulty.]

It might appear that the difficulty would be entirely avoided by abandoning Carnot's fundamental axiom; a view which is strongly urged by Mr. Joule (at the conclusion of his paper "On the Changes of Temperature produced by the Rarefaction and Condensation of Air." *Phil. Mag.*, May 1845, vol. xxvi.) If we do so, however, we meet with innumerable other difficulties—insuperable without farther experimental investigation, and an entire reconstruction of the theory of heat from its foundation. It is in reality to experiment that we must look—either for a verification of Carnot's axiom, and an explanation of the difficulty we have been considering; or for an entirely new basis of the Theory of Heat.

of the thermal agency is wasted. Hence it is of primary importance to discover the criterion of a perfect engine. This has been done by Carnot, who proves the following proposition :

13. *A perfect thermodynamic engine is such that, whatever amount of mechanical effect it can derive from a certain thermal agency, if an equal amount be spent in working it backwards, an equal reverse thermal effect will be produced.**

14. This proposition will be made clearer by the applications of it which are given later (§ 29), in the cases of the air-engine and the steam-engine, than it could be by any general explanation; and it will also appear, from the nature of the operations described in those cases, and the principles of Carnot's reasoning, that a perfect engine may be constructed with any substance of an indestructible texture as the alternately expanding and contracting medium. Thus we might conceive thermodynamic engines founded upon the expansions and contractions of a perfectly elastic solid, or of a liquid; or upon the alterations of volume experienced by substances in passing from the liquid to the solid state,† each of which being perfect, would

* For a demonstration, see § 29.

† A case minutely examined in another paper, to be laid before the Society at the present meeting. "Theoretical

produce the same amount of mechanical effect from a given thermal agency; but there are two cases which Carnot has selected as most worthy of minute attention, because of their peculiar appropriateness for illustrating the general principles of his theory, no less than on account of their very great practical importance: the steam-engine, in which the substance employed as the transferring medium is water, alternately in the liquid state and in the state of vapor; and the air-engine, in which the transference is effected by means of the alternate expansions and contractions of a medium always in the gaseous state. The details of an actually practicable engine of either kind are not contemplated by Carnot in his general theoretical reasonings, but he confines himself to the ideal construction, in the simplest possible way in each case, of an engine in which the economy is perfect. He thus determines the degree of perfectibility which cannot be surpassed; and by describing a conceivable method of attaining to this perfection by an air-engine or a steam-engine, he points out the proper objects to be kept in view in the practical construction and working of such machines. I now proceed to give an outline of these investigations.

Considerations on the Effect of Pressure in Lowering the Freezing-point of Water," by Prof. James Thomson.

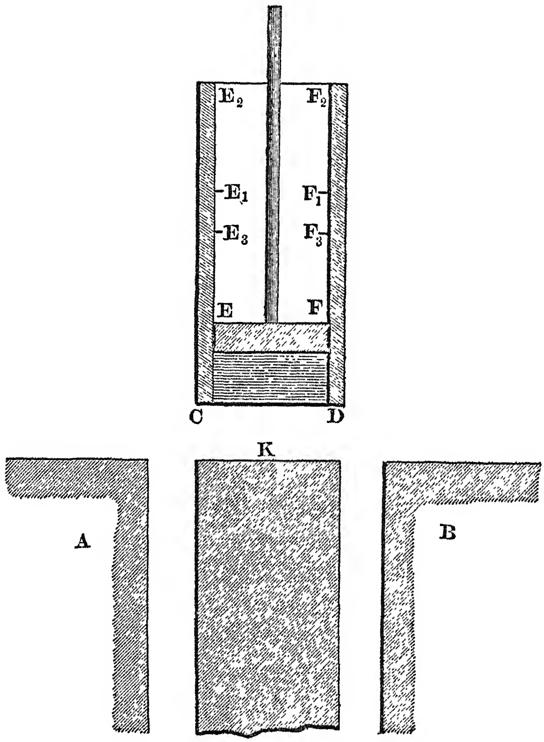
CARNOT'S THEORY OF THE STEAM-ENGINE.

15. Let CDF_2E_2 be a cylinder, of which the curved surface is perfectly impermeable to heat, with a piston also impermeable to heat, fitted in it; while the fixed bottom CD, itself with no capacity for heat, is possessed of perfect conducting power. Let K be an impermeable stand, such that when the cylinder is placed upon it the contents below the piston can neither gain nor lose heat. Let A and B be two bodies permanently retained at constant temperatures, $S°$ and $T°$, respectively, of which the former is higher than the latter. Let the cylinder, placed on the impermeable stand, K, be partially filled with water, at the temperature S, of the body A, and (there being no air below it) let the piston be placed in a position EF, near the surface of the water. The pressure of the vapor above the water will tend to push up the piston, and must be resisted by a force applied to the piston,* till

* In all that follows, the pressure of the atmosphere on the upper side of the piston will be included in the applied forces, which, in the successive operations described, are sometimes overcome by the upward motion, and sometimes yielded to in the motion downwards. It will be unnecessary, in reckoning at the end of a cycle of operations, to take into account the work thus spent upon the atmosphere, and the restitution which has been made, since these precisely compensate for one another.

the commencement of the operations, which are conducted in the following manner:

(1) The cylinder being placed on the body *A*,



so that the water and vapor may be retained at the temperature *S*, *let the piston rise any convenient*

height EE_1, to a position E_1F_1, performing work by the pressure of the vapor below it during its ascent.

[During this operation a certain quantity, H, of heat, the amount of latent heat in the fresh vapor which is formed, is abstracted from the body A.]

(2) The cylinder being removed, and placed on the impermeable stand K, *let the piston rise gradually, till, when it reaches a position E_2F_2, the temperature of the water and vapor is T, the same as that of the body B.*

[During this operation the fresh vapor continually formed requires heat to become latent; and, therefore, as the contents of the cylinder are protected from any accession of heat, their temperature sinks.]

(3) The cylinder being removed from K, and placed on B, *let the piston be pushed down, till, when it reaches the position E_3F_3, the quantity of heat evolved and abstracted by B amounts to that which, during the first operation, was taken from A.*

[Note of Nov. 5, 1881. The specification of this operation, with a view to the return to the primitive condition, intended as the conclusion to the four operations, is the only item in which Carnot's temporary and provisional assumption of the materiality of heat has effect. To exclude this hypothesis, Prof. James Thomson has suggested the fol-

lowing corrected specification for the third operation: *Let the piston be pushed down, till it reaches a position E_3F_3, determined so as to fulfil the condition, that at the end of the fourth operation the primitive temperature S shall be reached:**]

[During this operation the temperature of the contents of the cylinder is retained constantly at $T°$, and all the latent heat of the vapor which is condensed into water at the same temperature is given out to *B*.]

(4) The cylinder being removed from *B*, and placed on the impermeable stand, *let the piston be pushed down from E_3F_3 to its original position EF.*

[During this operation, the impermeable stand preventing any loss of heat, the temperature of the water and air must rise continually, till (since the quantity of heat evolved during the third operation was precisely equal to

* [Note of Nov. 5, 1881. Maxwell has simplified the correction by beginning the cycle with Carnot's second operation, and completing it through his third, fourth, and first operations, with his third operation nearly as follows:

let the piston be pushed down to any position E_3F_3;

then Carnot's fourth operation altered to the following:

let the piston be pushed down from E_3F_3 until the temperature reaches its primitive value S;

and lastly, Carnot's first operation altered to the following:

let the piston rise to its primitive position.]

that which was previously absorbed) at the conclusion it reaches its primitive value, S, in virtue of Carnot's fundamental axiom.]

[Note of Nov. 5, 1881. With Prof. James Thomson's correction of operation (3), the words in virtue of "Carnot's Fundamental Axiom" must be replaced by "the condition fulfilled by operation (3)," in the description of the results of operation (4).]

16. At the conclusion of this cycle of operations* the total thermal agency has been the *letting down* of H units of heat from the body A, at the temperature S, to B, at the lower temperature T; and the aggregate of the mechanical effect has been a certain amount of *work produced*, since during the ascent of the piston in the first and second operations, the temperature of the water and vapor, and therefore the pressure of the vapor on the piston, was on the whole higher than during the descent, in the third and fourth operations. It remains for us actually to evaluate this aggregate amount of work performed; and for this purpose the follow-

* In Carnot's work some perplexity is introduced with reference to the temperature of the water, which, in the operations he describes, is not brought back exactly to what it was at the commencement; but the difficulty which arises is explained by the author. No such difficulty occurs with reference to the cycle of operation described in the text, for which I am indebted to Mons. Clapeyron.

ing graphical method of representing the mechanical effect developed in the several operations, taken from Mons. Clapeyron's paper, is extremely convenient.

17. Let OX and OY be two lines at right angles to one another. Along OX measure off distances ON_1, N_1N_2, N_2N_3, N_3O, respectively proportional to the spaces described by the piston during the four successive operations described above; and, with reference to these four operations respectively, let the following constructions be made:

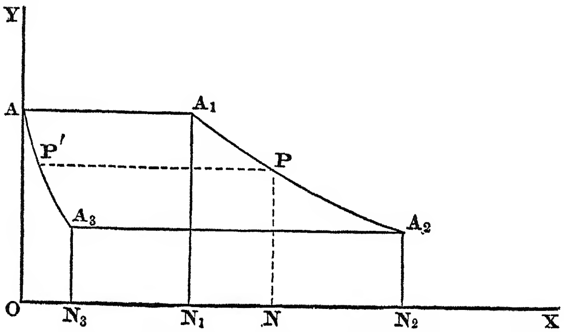
(1) Along OY measure a length OA, to represent the pressure of the saturated vapor at the temperature S; and draw AA_1 parallel to OX, and let it meet an ordinate through N_1, in A_1.

(2) Draw a curve A_1PA such that, if ON represent, at any instant during the second operation, the distance of the piston from its primitive position, NP shall represent the pressure of the vapor at the same instant.

(3) Through A_2 draw A_2A_3 parallel to OX, and let it meet an ordinate through N_3 in A_3.

(4) Draw the curve A_3A such that the abscissa and ordinate of any point in it may represent respectively the distances of the piston from its primitive position, and the pressure of the vapor, at each instant during the fourth operation. The

last point of this curve must, according to Carnot's fundamental principle, coincide with A, since the piston is, at the end of the cycle of operations,



again in its primitive position, and the pressure of the vapor is the same as it was at the beginning.

18. Let us now suppose that the lengths, ON_1, N_1N_2, N_2N_3, and N_3O, *represent numerically* the volumes of the spaces moved through by the piston during the successive operations. It follows that the mechanical effect obtained during the first operation will be *numerically represented* by the area AA_1N_1O; that is, the number of superficial units in this area will be equal to the number of "foot-pounds" of work performed by the ascending piston during the first operation. The work performed by the piston during the second operation will be similarly represented by the area

$A_1 A_2 N_2 N_1$. Again, during the third operation a certain amount of work is spent on the piston, which will be represented by the area $A_2 A_3 N_3 N_2$; and lastly, during the fourth operation, work is spent in pushing the piston to an amount represented by the area $A_3 A O N_3$.

19. Hence the mechanical effect (represented by the area $OAA\ A_2 N_2$) which was obtained during the first and second operations, exceeds the work (represented by $N_2 A_2 A_3 A\ O$) spent during the third and fourth, by an amount represented by the area of the quadrilateral figure $AA_1 A_2 A_3$; and, consequently, it only remains for us to evaluate this area, that we may determine the total mechanical effect gained in a complete cycle of operations. Now, from experimental data, at present nearly complete, as will be explained below, we may determine the length of the line AA_1 for the given temperature S, and a given absorption H, of heat, during the first operation; and the length of $A_2 A_3$ for the given lower temperature T, and the evolution of the same quantity of heat during the fourth operation: and the curves $A_1 P A_2$, $A_3 P' A$ may be drawn as graphical representations of actual observations. The figure being thus constructed, its area may be measured, and we are, therefore, in possession of a graphical

method of determining the amount of mechanical effect to be obtained from any given thermal agency. As, however, it is merely the area of the figure which it is required to determine, it will not be necessary to be able to describe each of the curves A_1PA_2, $A_3P'A$, but it will be sufficient to know the difference of the abscissas corresponding to any equal ordinates in the two; and the following analytical method of completing the problem is the most convenient for leading to the actual numerical results.

20. Draw any line PP' parallel to OX, meeting the curvilinear sides of the quadrilateral in P and P'. Let ξ denote the length of this line, and p its distance from OX. The area of the figure, according to the integral calculus, will be denoted by the expression

$$\int_{p_3}^{p_1} \xi dp,$$

where p_1 and p_3 (the limits of integration indicated according to Fourier's notation) denote the lines OA and N_3A_3, which represent respectively the pressures during the first and third operations. Now, by referring to the construction described above, we see that ξ is the difference of the volumes below the piston at corresponding instants of the second and fourth operations, or instants at which

the saturated steam and the water in the cylinder have the same pressure p, and consequently the same temperature, which we may denote by t. Again, throughout the second operation the entire contents of the cylinder possess a greater amount of heat by H units than during the fourth; and, therefore, at any instant of the second operation there is as much more steam as contains H units of latent heat than at the corresponding instant of the fourth operation. Hence if k denote the latent heat in a unit of saturated steam at the temperature t, the volume of the steam at the two corresponding instants must differ by $\frac{H}{k}$. Now, if σ denote the ratio of the density of the steam to that of the water, the volume $\frac{H}{k}$ of steam will be formed from the volume $\sigma\frac{H}{k}$ of water; and consequently we have, for the difference of volumes of the entire contents at the corresponding instants,

$$\xi = (1 - \sigma)\frac{H}{k}.$$

Hence the expression for the area of the quadrilateral figure becomes

$$\int_{p_2}^{p_1} (1 - \sigma)\frac{H}{k}dp.$$

Now, σ, k, and p, being quantities which depend upon the temperature, may be considered as functions of t; and it will be convenient to modify the integral so as to make t the independent variable. The limits will be from $t = T$ to $t = S$, and, if we denote by M the value of the integral, we have the expression

$$M = H\int_T^S (1-\sigma)\frac{\frac{dp}{dt}}{k}\,dt. \quad . \quad . \quad . \quad (1)$$

for the total amount of mechanical effect gained by the operations described above.

21. If the interval of temperatures be extremely small,—so small that $(1-\sigma)\frac{\frac{dp}{dt}}{k}$ will not sensibly vary for values of t between T and S,—the preceding expression becomes simply

$$M = (1-\sigma)\frac{\frac{dp}{dt}}{k}.H(S-T). \quad . \quad . \quad (2)$$

This might, of course, have been obtained at once by supposing the breadth of the quadrilateral figure AA_1A_2A to be extremely small compared with its length, and then taking for its area, as an approximate value, the product of the breadth into

the line AA_1, or the line A_3A_2, or any line of intermediate magnitude.

The expression (2) is rigorously correct for any interval $S-T$, if the mean value of $(1-\sigma)\frac{\frac{dp}{dt}}{k}$ for that interval be employed as the coefficient of $H(S-T)$.

Carnot's Theory of the Air-engine.

22. In the ideal air-engine imagined by Carnot four operations performed upon a mass of air or gas enclosed in a closed vessel of variable volume constitute a complete cycle, at the end of which the medium is left in its primitive physical condition; the construction being the same as that which was described above for the steam-engine, a body A, permanently retained at the temperature S, and B at the temperature T; an impermeable stand K; and a cylinder and piston, which in this case contains a mass of air at the temperature S, instead of water in the liquid state, at the beginning and end of a cycle of operations. The four successive operations are conducted in the following manner:

(1) The cylinder is laid on the body A, so that the air in it is kept at the temperature S; and the piston is allowed to rise, performing work.

(2) The cylinder is placed on the impermeable stand K, so that its contents can neither gain nor lose heat, and the piston is allowed to rise farther, still performing work, till the temperature of the air sinks to T.

(3) The cylinder is placed on B, so that the air is retained at the temperature T, and the piston is pushed down till the air gives out to the body B as much heat as it had taken in from A, during the first operation.

[Note of Nov. 5, 1881. To eliminate the assumption of the materiality of heat, make Professor James Thomson's correction here also; as above in § 15; or take Maxwell's rearrangement of the cycle described in the foot-note to § 15, p. 144.]

(4) The cylinder is placed on K, so that no more heat can be taken in or given out, and the piston is pushed down to its primitive position.

23. *At the end of the fourth operation the temperature must have reached its primitive value S, in virtue of* CARNOT'S *axiom.*

24. Here, again, as in the former case, we observe that work is performed by the piston during the first two operations; and during the third and fourth work is spent upon it, but to a less amount, since the pressure is on the whole less during the third and fourth operations than during the first

and second, on account of the temperature being lower. Thus, at the end of a complete cycle of operations, mechanical effect has been obtained; and the thermal agency from which it is drawn is the taking of a certain quantity of heat from A, and *letting it down*, through the medium of the engine, to the body B at a lower temperature.

25. To estimate the actual amount of effect thus obtained, it will be convenient to consider the alterations of volume of the mass of air in the several operations as extremely small. We may afterwards pass by the integral calculus, or, practically, by summation to determine the mechanical effect whatever be the amplitudes of the different motions of the piston.

26. Let dq be the quantity of heat absorbed during the first operation, which is evolved again during the third; and let dv be the corresponding augmentation of volume which takes place while the temperature remains constant, as it does during the first operation.* The diminution of volume

* Thus, $\frac{dq}{dv}$ will be the partial differential coefficient, with respect to v, of that function of v and t which expresses the quantity of heat that must be added to a mass of air when in a "standard" state (such as at the temperature zero, and under the atmospheric pressure), to bring it to the temperature t and the volume v. That there is such a

in the third operation must be also equal to dv, or only differ from it by an infinitely small quantity of the second order. During the second operation we may suppose the volume to be increased by an infinitely small quantity ϕ; which will occasion a diminution of pressure and a diminution of temperature, denoted respectively by ω and τ. During the fourth operation there will be a diminution of volume and an increase of pressure and temperature, which can only differ, by infinitely small quantities of the second order, from the changes in the other direction, which took place in the second operation, and they also may, therefore, be denoted by ϕ, ω, and τ, respectively. The alteration of pressure

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function, of two independent variables v and t, is merely an analytical expression of Carnot's fundamental axiom, as applied to a mass of air. The general principle may be analytically stated in the following terms :—If Mdv denote the accession of heat received by a mass of any kind, not possessing a destructible texture, when the volume is increased by dv, the temperature being kept constant, and if Ndt denote the amount of heat which must be supplied to raise the temperature by dt, without any alteration of volume; then $Mdv + Ndt$ must be the differential of a function of v and t. [Note of Nov. 5, 1881. In the corrected theory it is $(M - Jp)\,dv + Ndt$, that is a complete differential, not $Mdv + Ndt$. See *Dynamical Theory of Heat* (Art. XLVIII., below), § 20.]

during the first and third operations may at once be determined by means of Mariotte's law, since in them the temperature remains constant. Thus, if, at the commencement of the cycle, the volume and pressure be v and p, they will have become $v + dv$ and $pv/(v + dv)$ at the end of the first operation. Hence the diminution of pressure during the first operation is $p - pv/(v + dv)$ or $pdv/(v + dv)$ and therefore, if we neglect infinitely small quantities of the second order, we have pdv/v for the diminution of pressure during the first operation; which to the same degree of approximation, will be equal to the increase of pressure during the third. If $t + \tau$ and t be taken to denote the superior and inferior limits of temperature, we shall thus have for the volume, the temperature, and the pressure at the commencements of the four successive operations, and at the end of the cycle, the following values respectively:

$$
\begin{array}{llll}
(1) & v, & t + \tau, & p; \\
(2) & v + dv, & t + \tau, & p\left(1 - \dfrac{dv}{v}\right); \\
(3) & v + dv + \phi, & t, & p\left(1 - \dfrac{dv}{v}\right) - \omega; \\
(4) & v + \phi, & t, & p - \omega; \\
(5) & v, & t + \tau, & p.
\end{array}
$$

Taking the mean of the pressures at the beginning and end of each operation, we find

$$(1) \qquad p\left(1-\frac{1}{2}\frac{dv}{v}\right),$$

$$(2) \qquad p\left(1-\frac{dv}{v}\right)-\frac{1}{2}\omega,$$

$$(3) \qquad p\left(1-\frac{1}{2}\frac{dv}{v}\right)-\omega,$$

$$(4) \qquad p-\frac{1}{2}\omega,$$

which, as we are neglecting infinitely small quantities of the second order, will be the expressions for the mean pressures during the four successive operations. Now, the mechanical effect gained or spent, during any of the operations, will be found by multiplying the mean pressure by the increase or diminution of volume which takes place; and we thus find

$$(1) \qquad p\left(1-\frac{1}{2}\frac{dv}{v}\right)dv,$$

$$(2) \qquad \left\{p\left(1-\frac{dv}{v}\right)-\frac{1}{2}\omega\right\}\phi,$$

$$(3) \qquad \left\{p\left(1-\frac{1}{2}\frac{dv}{v}\right)-\omega\right\}dv,$$

$$(4) \qquad \left(p-\frac{1}{2}\omega\right)\phi.$$

for the amounts gained during the first and second, and spent during the third and fourth operations; and hence, by addition and subtraction, we find

$$\omega dv - p\phi \frac{dv}{v}, \quad \text{or} \quad (v\omega - p\phi)\frac{dv}{v},$$

for the aggregate amount of mechanical effect gained during the cycle of operations. It only remains for us to express this result in terms of dq and τ, on which the given thermal agency depends. For this purpose we remark that ϕ and ω are alterations of volume and pressure which take place along with a change of temperature τ, and hence, by the laws of compressibility and expansion, we may establish a relation* between them in the following manner:

Let p_0 be the pressure of the mass of air when reduced to the temperature zero, and confined in a volume v_0; then, whatever be v_0, the product $p_0 v_0$ will, by the law of compressibility, remain constant; and, if the temperature be elevated from 0 to $t + \tau$, and the gas be allowed to expand freely without any change of pressure, its volume will be

* We might also investigate another relation, to express the fact that there is no accession or removal of heat during either the second or the fourth operation; but it will be seen that this will not affect the result in the text, although it would enable us to determine both ϕ and ω in terms of τ.

increased in the ratio of 1 to $1 + E(t + \tau)$, where E is very nearly equal to .00366 (the Centigrade scale of the air-thermometer being referred to), whatever be the gas employed, according to the researches of Regnault and of Magnus on the expansion of gases by heat. If, now, the volume be altered arbitrarily with the temperature continually at $t + \tau$, the product of the pressure and volume will remain constant; and therefore we have

$$pv = p_0 v_0 \{1 + E(t + \tau)\}.$$

Similarly,

$$(p - \omega)(v + \phi) = p_0 v_0 \{1 + Et\}.$$

Hence, by subtraction, we have

$$v\omega - p\phi + \omega\phi = p_0 v_0 E\tau,$$

or, neglecting the product $\omega\phi$,

$$v\omega - p\phi = p_0 v_0 E\tau.$$

Hence the preceding expression for mechanical effect, gained in the cycle of operations, becomes

$$p_0 v_0 \,.\, E\tau \,.\, dv/v.$$

Or, as we may otherwise express it,

$$\frac{Ep_0 v_0}{v\,dq/dv} \,.\, dq \,.\, \tau.$$

Hence, if we denote by M the mechanical effect due to H units of heat descending through the same interval τ, which might be obtained by repeating

the cycle of operations described above, $\frac{H}{dq}$ times, we have

$$M = \frac{Ep_0v_0}{vdq/dv} \, . \, H\tau. \quad . \quad . \quad . \quad . \qquad (3)$$

27. If the *amplitudes* of the operations had been finite, so as to give rise to an absorption of H units of heat during the first operation, and a lowering of temperature from S to T during the second, the amount of work obtained would have been found to be expressed by means of a double definite integral thus:*

$$\left.\begin{aligned} M &= \int_0^H dq \int_T^S dt \, . \, \frac{Ep_0v_0}{vdq/dv}, \\ \text{or} \qquad M &= Ep_0v_0 \int_0^H \int_T^S \frac{1}{v} \frac{dv}{dq} \, . \, dtdq; \end{aligned}\right\} \, . \quad (4)$$

this second form being sometimes more convenient.

* This result might have been obtained by applying the usual notation of the integral calculus to express the area of the curvilinear quadrilateral, which, according to Clapeyron's graphical construction, would be found to represent the entire mechanical effect gained in the cycle of operations of the air-engine. It is not necessary, however, to enter into the details of this investigation, as the formula (3), and the consequences derived from it, include the whole theory of the air-engine, in the best practical form; and the investigation of it which I have given in the text will probably give as clear a view of the reasoning on which it is founded as could be obtained by the graphical method, which in this case is not so valuable as it is from its simplicity in the case of the steam-engine.

28. The preceding investigations, being founded on the approximate laws of compressibility and expansion (known as the law of Mariotte and Boyle, and the law of Dalton and Gay-Lussac), would require some slight modifications to adapt them to cases in which the gaseous medium employed is such as to present sensible deviations from those laws. Regnault's very accurate experiments show that the deviations are insensible, or very nearly so, for the ordinary gases at ordinary pressures; although they may be considerable for a medium, such as sulphurous acid, or carbonic acid under high pressure, which approaches the physical condition of a vapor at saturation; and therefore, in general, and especially in practical applications to real air-engines, it will be unnecessary to make any modification in the expressions. In cases where it may be necessary, there is no difficulty in making the modifications, when the requisite data are supplied by experiment.

29.* Either the steam-engine or the air-engine, according to the arrangements described above, gives all the mechanical effect that can possibly be obtained from the thermal agency employed. For

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* This paragraph is the demonstration, referred to above, of the proposition stated in § 13, as it is readily seen that it is applicable to any conceivable kind of thermodynamic engine.

it is clear that in either case the operations may be performed in the reverse order, with every thermal and mechanical effect reversed. Thus, in the steam-engine, we may commence by placing the cylinder on the impermeable stand, allow the piston to rise, performing work, to the position E_3F_3; we may then place it on the body B, and allow it to rise, performing work, till it reaches E_2F_2; after that the cylinder may be placed again on the impermeable stand, and the piston may be pushed down to E_1F_1; and, lastly, the cylinder being removed to the body A, the piston may be pushed down to its primitive position. In this inverse cycle of operations a certain amount of work has been spent, precisely equal, as we readily see, to the amount of mechanical effect gained in the direct cycle described above; and heat has been abstracted from B, and deposited in the body A, at a higher temperature, to an amount precisely equal to that which in the direct style was *let down* from A to B. Hence it is impossible to have an engine which will derive more mechanical effect from the same thermal agency than is obtained by the arrangement described above; since, if there could be such an engine, it might be employed to perform, as a part of its whole work, the inverse cycle of operations, upon an engine of the

kind we have considered, and thus to continually restore the heat from B to A, which has descended from A to B for working itself; so that we should have a complex engine, giving a residual amount of mechanical effect without any thermal agency, or alteration of materials, which is an impossibility in nature. The same reasoning is applicable to the air-engine; and we conclude, generally, that any two engines, constructed on the principles laid down above, whether steam-engines with different liquids, an air-engine and a steam-engine, or two air-engines with different gases, must derive the same amount of mechanical effect from the same thermal agency.

30. Hence, by comparing the amounts of mechanical effect obtained by the steam-engine and the air-engine from the letting down of the H units of heat from A at the temperature $(t + \tau)$ to B at t, according to the expressions (2) and (3), we have

$$M = (1 - \sigma) \frac{dp}{kdt} \cdot H\tau = \frac{Ep_0v_0}{vdq/dv} \cdot H\tau. \quad . \quad (5)$$

If we denote the coefficient of $H\tau$ in these equal expressions by μ, which may be called "Carnot's coefficient," we have

$$\mu = (1 - \sigma) \frac{dp}{kdt} = \frac{Ep_0v_0}{vdq/dv}, \quad . \quad . \quad (6)$$

and we deduce the following very remarkable conclusions:

(1) For the saturated vapors of all different liquids, at the same temperature, the value of $(1 - \sigma)\frac{dp}{kdt}$ must be the same.

(2) For any different gaseous masses, at the same temperature, the value of $\frac{Ep_0v_0}{vdq/dv}$ must be the same.

(3) The values of these expressions for saturated vapors and for gases, at the same temperature, must be the same.

31. No conclusion can be drawn *a priori* regarding the values of this coefficient μ for different temperatures, which can only be determined, or compared, by experiment. The results of a great variety of experiments, in different branches of physical science (Pneumatics and Acoustics), cited by Carnot and by Clapeyron, indicate that the values of μ for low temperatures exceed the values for higher temperatures; a result amply verified by the continuous series of experiments performed by Regnault on the saturated vapor of water for all temperacures from 0° to 230°, which, as we shall see later, give values for μ gradually diminishing from the inferior limit to the superior limit of

temperature. When, by observation, μ has been determined as a function of the temperature, the amount of mechanical effect, M, deducible from H units of heat descending from a body at the temperature S to a body at the temperature T, may be calculated from the expression

$$M = H \int_S^T \mu dt, \quad . \quad . \quad . \quad . \quad (7)$$

which is, in fact. what either of the equations (1) for the steam-engine, or (4) for the air-engine, becomes, when the notation μ, for Carnot's multiplier, is introduced.

The values of this integral may be practically obtained, in the most convenient manner, by first determining, from observation, the mean values of μ for the successive degrees of the thermometric scale, and then adding the values for all the de grees within the limits of the extreme temperatures S and T.*

32. The complete theoretical investigation of the motive power of heat is thus reduced to the experimental determination of the coefficient μ; and may be considered as perfect, when, by any series of experimental researches whatever, we can

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* The results of these investigations are exhibited in Tables I and II.

find a value of μ for every temperature within practical limits. The special character of the experimental researches, whether with reference to gases or with reference to vapors, necessary and sufficient for this object, is defined and restricted in the most precise manner, by the expressions (6) for μ, given above.

33. The object of Regnault's great work, referred to in the title of this paper, is the experimental determination of the various physical elements of the steam-engine; and when it is complete, it will furnish all the *data* necessary for the calculation of μ. The valuable researches already published in a first part of that work make known the latent heat of a given weight, and the pressure, of saturated steam for all temperatures between 0° and 230° Cent. of the air-thermometer. Besides these data, however, the density of saturated vapor must be known, in order that k, the latent heat of a unit of volume, may be calculated from Regnault's determination of the latent heat of a given weight.* Between the limits of 0° and 100°,

* It is, comparatively speaking, of little consequence to know accurately the value of σ, for the factor $(1-\sigma)$ of the expression for μ, since it is so small (being less than $\frac{1}{1700}$ for all temperatures between 0° and 100°) that, unless all the data are known with more accuracy than we can

it is probable, from various experiments which have been made, that the density of vapor follows very closely the simple laws which are so accurately verified by the ordinary gases;* and thus it may be calculated from Regnault's table giving the pressure at any temperature within those limits. Nothing as yet is known with accuracy as to the density of saturated steam between 100 and 230°, and we must be contented at present to estimate it by calculation from Regnault's table of pressures; although, when accurate experimental researches on the subject shall have been made, considerable deviations from the laws of Boyle and Dalton, on which this calculation is founded, may be discovered.

34. Such are the experimental data on which the mean values of μ for the successive degrees of the air-thermometer, from 0 to 230°, at present laid before the Royal Society, is founded. The unit of length adopted is the English foot; the unit of weight, the pound; the unit of work, a

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count upon at present, we might neglect it altogether, and take dp/kdt simply, as the expression for μ, without committing any error of important magnitude.

* This is well established, within the ordinary atmospheric limits, in Regnault's Études Météorologiques, in the *Annales de Chimie*, vol. xv., 1846.

"foot-pound;" and the unit of heat that quantity which, when added to a pound of water at 0°, will produce an elevation of 1° in temperature. The mean value of μ for any degree is found to a sufficient degree of approximation by taking, in place of σ, dp/dt and k; in the expression

$$(1-\sigma)\cdot\frac{dp}{kdt};$$

the mean values of those elements; or, what is equivalent to the corresponding accuracy of approximation, by taking, in place of σ and k respectively, the mean of the values of those elements for the limits of temperature, and in place of dp/dt, the difference of the values of p, at the same limits.

35. In Regnault's work (at the end of the eighth memoir), a table of the pressures of saturated steam for the successive temperatures 0°, 1°, 2°,... 230°, expressed in millimetres of mercury, is given. On account of the units adopted in this paper, these pressures must be estimated in pounds on the square foot, which we may do by multiplying each number of millimetres by 2.7896, the weight in pounds of a sheet of mercury, one millimetre thick, and a square foot in area.

36. The value of k, the latent heat of a cubic foot, for any temperature t, is found from λ, the

latent heat of a pound of saturated steam, by the equation

$$k = \frac{p}{760} \cdot \frac{1 + .00366 \times 100}{1 + .00366 \times t} \cdot \times .036869^{*} . \lambda,$$

where p denotes the pressure in millimetres, and λ the latent heat of a pound of saturated steam; the values of λ being calculated by the empirical formula †

$$\lambda = (606.5 + 0.305t) - (t + .00002t^2 + 0.0000003t^3),$$

given by Regnault as representing, between the

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* It appears that the vol. of 1 kilog. must be 1.69076 according to the data here assumed.

The density of saturated steam at 100° is taken as $\frac{1}{1693.5}$ of that of water at its maximum. Rankine takes it as $\frac{1}{1696}$.

† The part of this expression in the first vinculum (see Regnault, end of ninth memoir) is what is known as "the total heat" of a pound of steam, or the amount of heat necessary to convert a pound of water at 0° into a pound of saturated steam at $t°$; which, according to "Watt's law," thus approximately verified, would be constant. The second part, which would consist of the single term t, if the specific heat of water were constant for all temperatures, is the number of thermic units necessary to raise the temperature of a pound of water from 0° to $t°$, and expresses empirically the results of Regnault's experiments on the specific heat of water (see end of the tenth memoir), described in the work already referred to.

extreme limits of his observations, the latent heat of a unit weight of saturated steam.

Explanation of Table I.

37. The mean values of μ for the first, for the eleventh, for the twenty-first, and so on, up to the 231st* degree of the air-thermometer, have been calculated in the manner explained in the preceding paragraphs. These, and interpolated results, which must agree with what would have been obtained, by direct calculation from Regnault's data, to three significant places of figures (and even for the temperatures between 0° and 100°, the experimental data do not justify us in relying on any of the results to a greater degree of accuracy), are exhibited in Table I.

To find the amount of mechanical effect due to a unit of heat, descending from a body at a temperature S to a body at T, if these numbers be integers, we have merely to add the values of μ in Table I. corresponding to the successive numbers.

$$T+1,\ T+2,\ \ldots .\ S-2,\ S-1.$$

* In strictness, the 230th is the last degree for which the experimental data are complete; but the data for the 231st may readily be assumed in a sufficiently satisfactory manner.

Explanation of Table II.

38. The calculation of the mechanical effect, in any case, which might always be effected in the manner described in § 37 (with the proper modification for fractions of degrees, when necessary), is much simplified by the use of Table II., where the first number of Table I., the sum of the first and second, the sum of the first three, the sum of the first four, and so on, are successively exhibited. The sums thus tabulated are the values of the integrals

$$\int_0^1 \mu dt, \quad \int_0^2 \mu dt, \quad \int_0^3 \mu dt, \ldots . \int_0^{231} \mu dt;$$

and, if we denote $\int_0^t \mu dt$ by the letter M, Table II. may be regarded as a table of the value of M.

To find the amount of mechanical effect due to a unit of heat descending from a body at a temperature S to a body at T, if these numbers be integers, we have merely to subtract the value of M, for the number T, from the value for the number S, given in Table II

TABLE I.*

MEAN VALUES OF μ FOR THE SUCCESSIVE DEGREES OF THE AIR-THERMOMETER FROM 0° TO 230°.

°	μ	°	μ	°	μ	°	μ
1	4.960	32	4.559	63	4.194	94	3.889
2	4.946	33	4.547	64	4.183	95	3.880
3	4.932	34	4.535	65	4.172	96	3.871
4	4.918	35	4.522	66	4.161	97	3.863
5	4.905	36	4.510	67	4.150	98	3.854
6	4.892	37	4 498	68	4.140	99	3.845
7	4.878	38	4.486	69	4.129	100	3.837
8	4.865	39	4.474	70	4.119	101	3.829
9	4.852	40	4.462	71	4.109	102	3.820
10	4.839	41	4.450	72	4.098	103	3.812
11	4.826	42	4.438	73	4.088	104	3.804
12	4.812	43	4.426	74	4.078	105	3.796
13	4.799	44	4.414	75	4.067	106	3.788
14	4.786	45	4.402	76	4.057	107	3.780
15	4.773	46	4.390	77	4.047	108	3.772
16	4.760	47	4 378	78	4.037	109	3.764
17	4.747	48	4.366	79	4.028	110	3.757
18	4.735	49	4.355	80	4.018	111	3.749
19	4.722	50	4.343	81	4.009	112	3.741
20	4.709	51	4.331	82	3.999	113	3.734
21	4.697	52	4.319	83	3.990	114	3.726
22	4.684	53	4.308	84	3.980	115	3.719
23	4.672	54	4.296	85	3.971	116	3.712
24	4.659	55	4.285	86	3.961	117	3.704
25	4.646	56	4.273	87	3.952	118	3.697
26	4.634	57	4.262	88	3.943	119	3.689
27	4.621	58	4.250	89	3.934	120	3.682
28	4.609	59	4.239	90	3.925	121	3.675
29	4.596	60	4.227	91	3.916	122	3.668
30	4.584	61	4.216	92	3.907	123	3.661
31	4.572	62	4.205	93	3.898	124	3.654

* The numbers here tabulated may also be regarded as *the actual values of μ for* $t=\frac{1}{2}$, $t=1\frac{1}{2}$, $t=2\frac{1}{2}$, $t=3\frac{1}{2}$, etc.

TABLE I.—(*Continued.*)

°	μ	°	μ	°	μ	°	μ
125	3.647	152	3.479	179	3.342	206	3.225
126	3.640	153	3.473	180	3.337	207	3.221
127	3.633	154	3.468	181	3.332	208	3.217
128	3.627	155	3.462	182	3.328	209	3.213
129	3.620	156	3.457	183	3.323	210	3.210
130	3.614	157	3.451	184	3.318	211	3.206
131	3.607	158	3.446	185	3.314	212	3.202
132	3.601	159	3.440	186	3.309	213	3.198
133	3.594	160	3.435	187	3.304	214	3.195
134	3.586	161	3.430	188	3.300	215	3.191
135	3.579	162	3.424	189	3.295	216	3.188
136	3.573	163	3.419	190	3.291	217	3.184
137	3.567	164	3.414	191	3.287	218	3.180
138	3.561	165	3.409	192	3.282	219	3.177
139	3.555	166	3.404	193	3.278	220	3.173
140	3.549	167	3.399	194	3.274	221	3.169
141	3.543	168	3.394	195	3.269	222	3.165
142	3.537	169	3.389	196	3.265	223	3.162
143	3.531	170	3.384	197	3.261	224	3.158
144	3.525	171	3.380	198	3.257	225	3.155
145	3.519	172	3.375	199	3.253	226	3.151
146	3.513	173	3.370	200	3.249	227	3.148
147	3.507	174	3.365	201	3.245	228	3.144
148	3.501	175	3.361	202	3.241	229	3.141
149	3.495	176	3.356	203	3.237	230	3.137
150	3.490	177	3.351	204	3.233	231	3.134
151	3.484	178	3.346	205	3.229		

TABLE II.

MECHANICAL EFFECT IN FOOT-POUNDS DUE TO A THERMIC UNIT CENTIGRADE, PASSING FROM A BODY, AT ANY TEMPERATURE LESS THAN 230° TO A BODY AT 0°.

Superior Limit of Temperature.	Mechanical Effect.	Superior Limit of Temperature.	Mechanical Effect.	Superior Limit of Temperature.	Mechanical Effect.
°	Ft.-Pounds.	°	Ft.-Pounds.	°	Ft.-Pounds.
1	4.960	38	179.287	75	337.084
2	9.906	39	183.761	76	341.141
3	14.838	40	188.223	77	345.188
4	19.756	41	192.673	78	349.225
5	24.661	42	197.111	79	353.253
6	29.553	43	201.537	80	357.271
7	34.431	44	205.951	81	361.280
8	39.296	45	210.353	82	365.279
9	44.148	46	214.743	83	369.269
10	48.987	47	219.121	84	373 249
11	53.813	48	223.487	85	377.220
12	58.625	49	227.842	86	381.181
13	63.424	50	232.185	87	385.133
14	68.210	51	236.516	88	389.076
15	72.983	52	240.835	89	393.010
16	77.743	53	245.143	90	396.935
17	82.490	54	249.439	91	400.851
18	87.225	55	253.724	92	404.758
19	91.947	56	257.997	93	408.656
20	96.656	57	262.259	94	412.545
21	101.353	58	266.509	95	416.425
22	106.037	59	270.748	96	420.296
23	110.709	60	274.975	97	424 159
24	115.368	61	279.191	98	428.013
25	120.014	62	283.396	99	431.858
26	124.648	63	287.590	100	435.695
27	129.269	64	291.773	101	439.524
28	133.878	65	295.945	102	443.344
29	138.474	66	300.106	103	447.156
30	143.058	67	304.256	104	450.960
31	147.630	68	308.396	105	454.756
32	152.189	69	312.525	106	458.544
33	156.736	70	316.644	107	462.324
34	161.271	71	320.752	108	466.096
35	165.793	72	324.851	109	469.860
36	170.303	73	328.939	110	473.617
37	174.801	74	333.017	111	477.366

TABLE II.—(*Continued.*)

Superior Limit of Temperature.	Mechanical Effect.	Superior Limit of Temperature.	Mechanical Effect.	Superior Limit of Temperature.	Mechanical Effect.
°	Ft.-Pounds.	°	Ft.-Pounds.	°	Ft.-Pounds.
112	481.107	152	625.105	192	760.069
113	484.841	153	628.578	193	763.347
114	488.567	154	632.046	194	766.621
115	492.286	155	635.508	195	769.890
116	495.998	156	638.965	196	773.155
117	499.702	157	642.416	197	776.416
118	503.399	158	645.862	198	779.673
119	507.088	159	649.302	199	782.926
120	510.770	160	652.737	200	786.175
121	514.445	161	656.167	201	789.420
122	518.113	162	659.591	202	792.661
123	521.174	163	663.010	203	795.898
124	525.428	164	666.424	204	799.131
125	529.075	165	669.833	205	802.360
126	532.715	166	673.237	206	805.585
127	536.348	167	676.636	207	808.806
128	539.975	168	680.030	208	812.023
129	543.595	169	683.419	209	815.236
130	547.209	170	686.803	210	818.446
131	550.816	171	690.183	211	821.652
132	554.417	172	693.558	212	824.854
133	558.051	173	696.928	213	828.052
134	561.597	174	700.293	214	831.247
135	565.176	175	703.654	215	834.438
136	568.749	176	707.010	216	837.626
137	572.316	177	710.361	217	840.810
138	575.877	178	713.707	218	843.990
139	579.432	179	717.049	219	847.167
140	582.981	180	720.386	220	850.340
141	586.524	181	723.718	221	853.509
142	590.061	182	727.046	222	856.674
143	593.592	183	730.369	223	859.836
144	597.117	184	733.687	224	862.994
145	600.636	185	737.001	225	866.149
146	604.099	186	740.310	226	869.300
147	607.656	187	743.614	227	872.448
148	611.157	188	746.914	228	875.592
149	614.652	189	750.209	229	878.733
150	618.142	190	753.500	230	881.870
151	621.626	191	756.787	231	885.004

Note on the curves described in Clapeyron's graphical method of exhibiting Carnot's Theory of the Steam-Engine.

39. At any instant when the temperature of the water and vapor is t, during the fourth operation (see above, § 16, and suppose, for the sake of simplicity, that at the beginning of the first and at the end of the fourth operation the piston is absolutely in contact with the surface of the water), the latent heat of the vapor must be precisely equal to the amount of heat that would be necessary to raise the temperature of the whole mass, if in the liquid state, from t to S.* Hence, if v' denote the volume of the vapor, c the mean capacity for heat of a pound of water between the temperatures S

* For at the end of the fourth operation the whole mass is liquid, and at the temperature S. Now, this state might be arrived at by first compressing the vapor into water at the temperature t, and then raising the temperature of the liquid to S; and however this state may be arrived at, there cannot, on the whole, be any heat added to or subtracted from the contents of the cylinder, since, during the fourth operation, there is neither gain nor loss of heat. This reasoning is, of course, founded on Carnot's fundamental principle, which is tacitly assumed in the commonly-received ideas connected with "Watt's law," the "latent heat of steam," and "the total heat of steam."

and t, and W the weight of the entire mass, in pounds, we have

$$kv' = c(S - t)W.$$

Again, the circumstances during the second operation are such that the mass of liquid and vapor possesses H units of heat more than during the fourth; and consequently, at the instant of the second operation, when the temperature is t, the volume v of the vapor will exceed v' by an amount of which the latent heat is H, so that we have

$$v = v' + \frac{H}{k}.$$

40. Now, at any instant, the volume between the piston and its primitive position is less than the actual volume of vapor by the volume of the water evaporated. Hence, if x and x' denote the abscissæ of the curve at the instants of the second and fourth operations respectively, when the temperature is t, we have

$$x = v - \sigma v, \quad x' = v' - \sigma v',$$

and, therefore, by the preceding equations,

$$x = \frac{1 - \sigma}{k}\{H + c(S - t)W\}, \quad . \quad . \quad . \quad (a)$$

$$x' = \frac{1 - \sigma}{k}c(S - t)W. \quad . \quad . \quad . \quad . \quad . \quad (b)$$

These equations, along with $y = y' = p$, . . . (c)

enable us to calculate, from the data supplied by Regnault, the abscissa and ordinate for each of the curves described above (§ 17) corresponding to any assumed temperature t. After the explanations of §§ 33, 34, 35, 36, it is only necessary to add that c is a quantity of which the value is very nearly unity, and would be exactly so were the capacity of water for heat the same at every temperature as it is between 0° and 1°; and that the value of $c(S-t)$, for any assigned values of S and t, is found, by subtracting the number corresponding to t from the number corresponding to s, in the column headed *"Nombre des unités de chaleur abandonneés par un kilogramme d'eau en descendant de $T°$ à 0°,"* of the last table (at the end of the tenth memoir) of Regnault's work. By giving S the value 230°, and by substituting successively 220, 210, 200, etc., for t, values for x, y, x', y', have been found, which are exhibited in the table opposite.

Temperatures. t	Volumes to be described by the piston, to complete the fourth operation. x'	Volumes from the primitive position of the piston to those occupied at instants of the second operation. x	Pressures of saturated steam, in pounds on the square foot. $y=y'=p$
0°	1269. W	$x'+5.409.H$	12.832
10	639.6. W	$x'+2.847.H$	25.567
20	337.3. W	$x'+1.571.H$	48.514
30	185.5. W	$x'+.9062.H$	88.007
40	105.9. W	$x'+.5442.H$	153.167
50	62.62. W	$x'+.3392.H$	256.595
60	38.19. W	$x'+.2188.H$	415.070
70	21.94. W	$x'+.1456.H$	650.240
80	15.38. W	$x'+.09962.H$	989.318
90	10.09. W	$x'+.06994.H$	1465.80
100	6.744. W	$x'+.05026.H$	2120.11
110	4.578. W	$x'+.03688.H$	2999.87
120	3.141. W	$x'+.02758.H$	4160.10
130	2.176. W	$x'+.02098.H$	5663.70
140	1.519. W	$x'+.01625.H$	7581.15
150	1.058. W	$x'+.01271.H$	9990.26
160	0.7369. W	$x'+.01010.H$	12976.2
170	0.5085. W	$x'+.008116.H$	16630.7
180	0.3454. W	$x'+.006592.H$	21051.5
190	0.2267. W	$x'+.005406.H$	26341.5
200	0.1409. W	$x'+.004472.H$	32607.7
210	0.0784. W	$x'+.003729.H$	39960.7
220	0.3310. W	$x'+.003130.H$	48512.4
230	0	$x'+.002643.H$	58376.6

Appendix.

(Read April 30, 1849.)

41. In p. 30 some conclusions drawn by Carnot from his general reasoning were noticed; according to which it appears, that if the value of μ for

any temperature is known, certain information may be derived with reference to the saturated vapor of any liquid whatever, and, with reference to any gaseous mass, without the necessity of experimenting upon the specific medium considered. Nothing in the whole range of Natural Philosophy is more remarkable than the establishment of general laws by such a process of reasoning. We have seen, however, that doubt may exist with reference to the truth of the axiom on which the entire theory is founded, and it therefore becomes more than a matter of mere curiosity to put the inferences deduced from it to the test of experience. The importance of doing so was clearly appreciated by Carnot; and, with such data as he had from the researches of various experimenters, he tried his conclusions. Some very remarkable propositions which he derives from his theory coincide with Dulong and Petit's subsequently discovered experimental laws with reference to the heat developed by the compression of a gas; and the experimental verification is therefore in this case (so far as its accuracy could be depended upon) decisive. In other respects, the data from experiment were insufficient, although, so far as they were available as tests, they were confirmatory of the theory.

42. The recent researches of Regnault add im-

mensely to the experimental data available for this object, by giving us the means of determining with considerable accuracy the values of μ within a very wide range of temperature, and so affording a trustworthy standard for the comparison of isolated results at different temperatures, derived from observations in various branches of physical science.

In the first section of this Appendix the theory is tested, and shown to be confirmed by the comparison of the values of μ found above, with those obtained by Carnot and Clapeyron from the observations of various experimenters on air, and the vapors of different liquids. In the second and third sections some striking confirmations of the theory arising from observations by Dulong, on the specific heat of gases, and from Mr. Joule's experiments on the heat developed by the compression of air, are pointed out; and in conclusion, the actual methods of obtaining mechanical effect from heat are briefly examined with reference to their economy.

I. *On the values of μ derived by Carnot and Clapeyron from observations on Air, and on the Vapors of various liquids.*

43. In Carnot's work, pp. 80–82, the mean value of μ between 0° and 1° is derived from the

experiments of Delaroche and Berard on the specific heat of gases, by a process approximately equivalent to the calculation of the value of $\frac{Ep_0v_0}{vdq/dv}$ for the temperature $\frac{1}{2}°$. There are also, in the same work, determinations of the values of μ from observations on the vapors of alcohol and water; but a table given in M. Clapeyron's paper, of the values of μ derived from the data supplied by various experiments with reference to the vapors of ether, alcohol, water, and oil of turpentine, at the respective boiling-points of these liquids, affords us the means of comparison through a more extensive range of temperature. In the cases of alcohol and water, these results ought of course to agree with those of Carnot. There are, however, slight discrepancies which must be owing to the uncertainty of the experimental data.* In the opposite table, Carnot's results with reference to air, and Clapeyron's results with reference to the four different liquids, are exhibited, and compared with the values of μ which have been given

* Thus, from Carnot's calculations, we find, in the case of alcohol 4.035, and in the case of water 3.648, instead of 3.963 and 3.658, which are Clapeyron's results in the same cases.

Names of the Media.	Temperatures.	Values of μ.	Values of μ deduced from Regnault's Observations.	Differences.
	°	(Carnot)		
Air	0.5	4.377	4.960	.383
Sulphuric		(Clapeyron)		
Ether.....	(Boil. pt.) 35.5	4.478	4.510	.032
Alcohol....	78.8	3.963	4.030	.071
Water.....	100	3.658	3.837	.179
Essence of Turpentine.	156.8	3.530	3.449	−.081

above (Table I.) for the same temperatures, as derived from Regnault's observations on the vapor of water.

44. It may be observed that the discrepancies between the results founded on the experimental data supplied by the different observers with reference to water at the boiling-point, are greater than those which are presented between the results deduced from any of the other liquids, and water at the other temperatures; and we may therefore feel perfectly confident that the verification is complete to the extent of accuracy of the observations.* The considerable discrepancy presented

* A still closer agreement must be expected when more accurate experimental data are afforded with reference to the other media. Mons. Regnault informs me that he is

by Carnot's result deduced from experiments on air, is not to be wondered at when we consider the very uncertain nature of his data.

45. The fact of the gradual decrease of μ through a very extensive range of temperature, being indicated both by Regnault's continuous series of experiments and by the very varied experiment on different media, and in different branches of Physical Science, must be considered as a striking verification of the theory.

II. *On the Heat developed by the Compression of Air.*

46. Let a mass of air, occupying initially a given volume V, under a pressure P, at a temperature t, be compressed to a less volume V', and allowed to part with heat until it sinks to its primitive temperature t. The quantity of heat which is evolved may be determined, according to Carnot's theory, when the particular value of μ,

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engaged in completing some researches, from which we may expect, possibly before the end of the present year, to be furnished with all the data for five or six different liquids which we possess at present for water. It is therefore to be hoped that, before long, a most important test of the validity of Carnot's theory will be afforded.

corresponding to the temperature t, is known. For, by § 30, equation (6), we have

$$v \frac{dq}{dv} = \frac{Ep_0v_0}{\mu},$$

where dq is the quantity of heat absorbed, when the volume is allowed to increase from v to $v + dv$; or the quantity evolved by the reverse operation. Hence we deduce

$$dq = \frac{Ep_0v_0}{\mu} \frac{dv}{v}. \quad . \quad . \quad . \quad . \qquad (8)$$

Now, $\frac{Ep_0v_0}{\mu}$ is constant, since the temperature remains unchanged; and therefore we may at once integrate the second number. By taking it between the limits V' and V, we thus find

$$Q = \frac{Ep_0v_0}{\mu} \log \frac{V}{V'}^*, \quad . \quad . \quad . \qquad (9)$$

where Q denotes the required amount of heat evolved by the compression from V to P'. This expression may be modified by employing the equations $PV = P'V' = p_0v_0 (1 + Et)$; and we thus obtain

$$Q = \frac{EPV}{\mu(1 + Et)} \log \frac{V}{V'} = \frac{EP'V'}{\mu(1 + Et)} \log \frac{V}{V'}. \quad (10)$$

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* The *Napierian* logarithm of $\frac{V}{V'}$ is here understood.

From this result we draw the following conclusion:

47. *Equal volumes of all elastic fluids, taken at the same temperature and pressure, when compressed to smaller equal volumes, disengage equal quantities of heat.*

This extremely remarkable theorem of Carnot's was independently laid down as a probable experimental law by Dulong, in his "*Récherches sur la Chaleur Spécifique des Fluides Élastiques,*" and it therefore affords a most powerful confirmation of the theory.*

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* Carnot varies the statement of his theorem, and illustrates it in a passage, pp. 81, 82, of which the following is translation:

"*When a gas varies in volume without any change of temperature, the quantities of heat absorbed or evolved by this gas are in arithmetical progression, if the augmentation or diminutions of volume are in geometrical progression.*

"When we compress a litre of air maintained at the temperature 10°, and reduce it to half a litre, it disengages a certain quantity of heat. If, again, the volume be reduced from half a litre to a quarter of a litre, from a quarter to an eighth, and so on the quantities of heat successively evolved will be the same.

"If, in place of compressing the air, we allow it to expand to two litres, four litres, eight litres, etc., it will be necessary to supply equal quantities of heat to maintain the temperature always at the same degree."

48. In some very remarkable researches made by Mr. Joule upon the heat developed by the compression of air, the quantity of heat produced in different experiments has been ascertained with reference to the amount of work spent in the operation. To compare the results which he has obtained with the indications of theory, let us determine the amount of work necessary actually to produce the compression considered above.

49. In the first place, to compress the gas from the volume $v + dv$ to v, the work required is pdv, or, since

$$pv = p_0v_0(1 + Et),$$

$$p_0v_0(1 + Et)\frac{dv}{v}.$$

Hence, if we denote by W the total amount of work necessary to produce the compression from V to V', we obtain, by integration,

$$W = p_0v_0(1 + Et) \log \frac{V}{V'}.$$

Comparing this with the expression above, we find

$$\frac{W}{Q} = \frac{\mu(1 + Et)}{E} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

50. Hence we infer that—

(1) The amount of work necessary to produce a unit of heat by the compression of a gas is the same for all gases at the same temperature;

(2) And that the quantity of heat evolved in all circumstances, when the temperature of the gas is given, is proportional to the amount of work spent in the compression.

51. The expression for the amount of work necessary to produce a unit of heat is

$$\frac{\mu(1+Et)}{E},$$

and therefore Regnault's experiments on steam are available to enable us to calculate its value for any temperature. By finding the values of μ at 0°, 10°, 20°, etc., from Table I., and by substituting successively the values 0, 10, 20, etc., for t, the following results have been obtained:

TABLE OF THE VALUES OF $\frac{\mu(1+Et)}{E}$.

Work requisite to produce a unit of Heat by the compression of a Gas.	Temperature of the Gas.	Work requisite to produce a unit of Heat by the compression of a Gas.	Temperature of the Gas.
Ft.-pounds.	°	Ft.-pounds.	°
1357.1	0	1446.4	120
1368.7	10	1455.8	130
1379.0	20	1465.3	140
1388.0	30	1475.8	150
1395.7	40	1489.2	160
1401.8	50	1499.0	170
1406.7	60	1511.3	180
1412.0	70	1523.5	190
1417.6	80	1536.5	200
1424.0	90	1550.2	210
1430.6	100	1564.0	220
1438.2	110	1577.8	230

Mr. Joule's experiments were all conducted at temperatures from 50° to about 60° Fahr., or from 10° to 16° Cent.; and consequently, although some irregular differences in the results, attributable to errors of observation inseparable from experiments of such a very difficult nature, are presented, no regular dependence on the temperature is observable. From three separate series of experiments, Mr. Joule deduces the following numbers for the work, in foot-pounds, necessary to produce a thermic unit Fahrenheit by the compression of a gas.

820, 814, 760.

Multiplying these by 1.8, to get the corresponding number for a thermic unit Centigrade, we find

1476, 1465, and 1368.

The largest of these numbers is most nearly conformable with Mr. Joule's views of the relation between such experimental "equivalents," and others which he obtained in his electro-magnetic researches; but the smallest agrees almost perfectly with the indications of Carnot's theory; from which, as exhibited in the preceding table, we should expect, from the temperature in Mr. Joule's experiments, to find a number between 1369 and 1379 as the result.*

* [Note added Mar. 14, 1851: 772 is now the most probable, 1390 foot-pounds for 1° Cent.]

III. *On the Specific Heats of Gases.*

52. The following proposition is proved by Carnot as a deduction from his general theorem regarding the specific heats of gases.

The excess of specific heat under a constant pressure above the specific heat at a constant volume, is the same for all gases at the same temperature and pressure.*

53. To prove this proposition, and to determine an expression for the "excess" mentioned in its enunciation, let us suppose a unit of volume of a gas to be elevated in temperature by a small amount, τ. The quantity of heat required to do this will be $A\tau$, if A denote the specific heat at a constant volume. Let us next allow the gas to expand without going down in temperature, until its pressure becomes reduced to its primitive value. The expansion which will take place will be $\frac{E\tau}{1+Et}$, if the temperature be denoted by t; and hence, by (8), the quantity of heat that must be supplied, to prevent any lowering of temperature, will be

$$\frac{Ep_0v_0}{\mu}\cdot\frac{E\tau}{1+Et}, \quad \text{or} \quad \frac{E^2p}{\mu(1+Et)^2}\tau.$$

* Or the capacity of a unit of volume for heat.

Hence the total quantity added is equal to

$$A\tau + \frac{E^2p}{\mu(1+Et)^2}\tau.$$

But, since B denotes the specific heat under constant pressure, the quantity of heat requisite to bring the gas into this state, from its primitive condition, is equal to $B\tau$; and hence we have

$$B = A + \frac{E^2p}{\mu(1+Et)^2}. \quad . \quad . \quad . \quad (12)$$

IV. *Comparison of the Relative Advantages of the Air-engine and Steam-engine.*

54. In the use of water-wheels for motive power, the economy of the engine depends not only upon the excellence of its adaptation for actually transmitting any given quantity of water through it, and producing the equivalent of work, but upon turning to account the entire available fall; so, as we are taught by Carnot, the object of a thermodynamic engine is to economize in the best possible way the transference of all the heat evolved, from bodies at the temperature of the source, to bodies at the lowest temperature at which the heat can be discharged. With reference, then, to any engine of the kind, there will be two points to be considered:

(1) The extent of the *fall* utilized.

(2) The economy of the engine, with the fall which it actually uses.

55. In the first respect, the air-engine, as Carnot himself points out, has a vast advantage over the steam-engine; since the temperature of the hot part of the machine may be made very much higher in the air-engine than would be possible in the steam-engine, on account of the very high pressure produced in the boiler, by elevating the temperature of the water which it contains to any considerable extent above the atmospheric boiling-point. On this account a "perfect air-engine" would be a much more valuable instrument than a "perfect steam-engine." *

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* Carnot suggests a combination of the two principles, with air as the medium for receiving the heat at a very high temperature from the furnace; and a second medium, alternately in the state of saturated vapor and liquid water, to receive the heat, discharged at an intermediate temperature from the air, and transmit it to the coldest part of the apparatus. It is possible that a complex arrangement of this kind might be invented which would enable us to take the heat at a higher temperature, and discharge it at a lower temperature than would be practicable in any simple air-engine or simple steam-engine. If so, it would no doubt be equally possible, and perhaps more convenient, to employ steam alone, but to use it at a very high temperature not in contact with water in the hottest part of

Neither steam-engines nor air-engines, however, are nearly perfect; and we do not know in which of the two kinds of machine the nearest approach to perfection may be actually attained. The beautiful engine invented by Mr. Stirling of Galston may be considered as an excellent beginning for the air-engine;* and it is only necessary to compare this with Newcomen's steam-engine, and consider what Watt has effected, to give rise to the most sanguine anticipations of improvement.

V. *On the Economy of Actual Steam-engines.*

56. The steam-engine being universally employed at present as the means for deriving motive power from heat, it is extremely interesting to examine, according to Carnot's theory, the economy actually attained in its use. In the first

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the apparatus, instead of, as in the steam-engine, always in a saturated state.

* It is probably this invention to which Carnot alludes in the following passage: "Il a été fait, dit-on, tout recemment en Angleterre des essais heureux sur le développement de la puissance motrice par l'action de la chaleur sur l'air atmosphérique. Nous ignorons entièrement ne quoi ces essais ont consisté, si toutefois ils sont réels."

place we remark, that out of the entire "fall" from the temperature of the coals to that of the atmosphere it is only part—that from the temperature of the boiler to the temperature of the condenser—that is made available; while the very great fall from the temperature of the burning coals to that of the boiler, and the comparatively small fall from the temperature of the condenser to that of the atmosphere, are entirely lost as far as regards the mechanical effect which it is desired to obtain. We infer from this, that the temperature of the boiler ought to be kept as high as, according to the strength, is consistent with safety, while that of the condenser ought to be kept as nearly down at the atmospheric temperature as possible. To take the entire benefit of the actual fall, Carnot showed that the "principle of expansion" must be pushed to the utmost.*

* From this point of view, we see very clearly how imperfect is the steam-engine, even after all Watt's improvements. For to "push the principle of expansion to the utmost," we must allow the steam, before leaving the cylinder, to expand until its pressure is the same as that of the vapor in the condenser. According to "Watt's law," its temperature would then be the same as (actually a little above, as Regnault has shown) that of the condenser, and

57. To obtain some notion of the economy which has actually been obtained, we may take the alleged performances of the best Cornish engines, and some other interesting practical cases, as examples.*

(1) The engine of *the Fowey Consols mine* was reported, in 1845, to have given 125,089,000 foot-pounds of effect, for the consumption of one bushel or 94 lbs. of coals. Now the average amount evaporated from Cornish boilers, by one pound of coal, is $8\frac{1}{2}$ lbs. of steam; and hence for each pound of steam evaporated 156,556 foot-pounds of work are produced.

The pressure of the saturated steam in the boiler may be taken as $3\frac{1}{2}$ atmospheres;† and, conse-

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hence the steam-engine worked in this most advantageous way has in reality the very fault that Watt found in Newcomen's engine. This defect is partially remedied by Hornblower's system of using a separate expansion cylinder, an arrangement the advantages of which did not escape Carnot's notice, although they have not been recognized extensively among practical engineers, until within the last few years.

* I am indebted to the kindness of Professor Gordon of Glasgow for the information regarding the various cases given in the text.

† In different Cornish engines, the pressure in the boiler

quently, the temperature of the water will be 140°. Now (Regnault, end of Mémoire X.) the latent heat of a pound of saturated steam at 140° is 508, and since, to compensate for each pound of steam removed from the boiler in the working of the engine, a pound of water, at the temperature of the condenser, which may be estimated at 30°, is introduced from the hot-well; it follows that 618 units of heat are introduced to the boiler for each pound of water evaporated. But the work produced, for each pound of water evaporated, was found above to be 156,556 foot-pounds. Hence $\frac{156556}{618}$, or 253 foot-pounds, is the amount of work produced for each unit of heat transmitted through the Fowey Consols engine. Now in Table II. we find 583.0 as the theoretical effect due to a unit descending from 140° to 0°, and 143 as the effect due to a unit descending from 30° to 0°. The difference of these numbers, or 440,* is the number of foot-

is from 2½ to 5 atmospheres; and, therefore, as we find from Regnault's table of the pressure of saturated steam, the temperature of the water in the boiler must, in all of them, lie between 128° and 152°. For the better class of engines, the average temperature of the water in the boiler may be estimated at 140°, the corresponding pressure of steam being 3½ atmospheres.

* This number agrees very closely with the number

pounds of work that a *perfect* engine with its boiler at 140° and its condenser at 30° would produce for each unit of heat transmitted. Hence the Fowey Consols engine, during the experiments reported on, performed $\frac{253}{440}$ of its theoretical duty, or $57\frac{1}{2}$ per cent.

(2) The best duty on record, as performed by an engine at work (not for merely experimental purposes), is that of Taylor's engine, at the United Mines, which in 1840 worked regularly for several months at the rate of 98,000,000 foot-pounds for each bushel of coals burned. This is $\frac{98}{125}$, or .784 of the experimental duty reported in the case of the Fowey Consols engine. Hence the best useful work on record is at the rate of 198.3 foot-pounds for each unit of heat transmitted, and is $\frac{198.3}{440}$, or 45 per cent of the theoretical duty, on the supposition that the boiler is at 140° and the condenser at 30°.

(3) French engineers contract (in Lille, in 1847, for example) to make engines for mill-power which will produce 30,000 metre-pounds or 98,427 foot-pounds of work for each pound of steam used. If

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corresponding to the fall from 100° to 0°, given in Table II. Hence, the fall from 140° to 30° of the scale of the air-thermometer is equivalent, with reference to motive power, to the fall from 100° to 0°.

we divide this by 618, we find 159 foot-pounds for the work produced by each unit of heat. This is 36.1 per cent of 440, the theoretical duty.*

(4) English engineers have contracted to make engines and boilers which will require only $3\frac{1}{3}$ lbs. of the best coal per horse-power per hour. Hence in such engines each pound of coal ought to produce 565,700 foot-pounds of work, and if 7 lbs. of water be evaporated by each pound of coal, there would result 83,814 foot-pounds of work for each pound of water evaporated. If the pressure in the

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* It being assumed that the temperatures of the boiler and condenser are the same as those of the Cornish engines. If, however, the pressure be lower, two atmospheres, for instance, the numbers would stand thus: The temperature in the boiler would be only 121. Consequently, for each pound of steam evaporated, only 614 units of heat would be required; and therefore the work performed for each unit of heat transmitted would be 160.3 foot-pounds, which is *more* than according to the estimate in the text. On the other hand, the range of temperatures, or the fall utilized, is only from 131 to 30, instead of from 140 to 30°, and, consequently (Table II.), the theoretical duty for each unit of heat is only 371 foot-pounds. Hence, if the engine, to work according to the specification, requires a pressure of only 15 lbs. on the square inch (i.e., a total steam-pressure of two atmospheres), its performance is $\frac{160.3}{371}$, or 43.2 per cent of its theoretical duty.

boiler be $3\frac{1}{2}$ atmospheres (temperature 140°) the amount of work for each unit of heat will be found, by dividing this by 618, to be 130.7 foot-pounds, which is $\frac{130.7}{440}$ or 29.7 per cent of the theoretical duty.*

(5) The actual average of work performed by good Cornish engines and boilers is 55,000,000 foot-pounds for each bushel of coal, or less than half the experimental performance of the Fowey Consols engine, more than half the actual duty performed by the United Mines engine in 1840; in fact, about 25 per cent of the theoretical duty.

(6) The average performances of a number of Lancashire engines and boilers have been recently found to be such as to require 12 lbs. of Lancashire coal per horse-power per hour (i.e., for performing $60 \times 33{,}000$ foot-pounds), and of a number of Glasgow engines such as to require 15 lbs. (of a somewhat inferior coal) for the same effect. There are, however, more than twenty large engines in Glasgow at present† which work with a

* If, in this case again, the pressure required in the boiler to make the engine work according to the contract were only 15 lbs. on the square inch, we should have a different estimate of the economy, for which see Table B, at the end of this paper.

† These engines are provided with separate expansion

consumption of only $6\frac{1}{2}$ lbs. of dross, equivalent to 5 lbs. of the best Scotch or 4 lbs. of the best Welsh coal, per horse-power per hour. The economy may be estimated from these data, as in the other cases, on the assumption which, with reference to these, is the most probable we can make, that the evaporation produced by a pound of best coal is 7 lbs. of steam.

58. The following tables afford a synoptic view of the performances and theoretical duties in the various cases discussed above.

In Table A the numbers in the second column are found by dividing the numbers in the first by $8\frac{1}{2}$ in cases (1), (2), and (5), and by 7 in cases (4), (6), and (7), the estimated numbers of pounds of steam actually produced in the different boilers by the burning of 1 lb. of coal.

The numbers in the third column are found from those in the second, by dividing by 618 in Table A, and 614 in Table B, which are respectively the quantities of heat required to convert a pound of water taken from the hot-well at 30°, into saturated steam, in the boiler, at 140° or at 121°.

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cylinders, which have been recently added to them by Mr. M'Naught of Glasgow.

With reference to the cases (3), (4), (6), (7), the hypothesis of Table B is probably in general nearer the truth than that of Table A. In (4), (6), and (7), especially upon hypothesis B, there is much uncertainty as to the amount of evaporation that will be actually produced by 1 lb. of fuel. The assumption on which the numbers in the second column in Table B are calculated, is, that each pound of coal will send the same number of units of heat into the boiler, whether hypothesis A or hypothesis B be followed. Hence, except in the case of the French contract, in which the *evaporation*, not the fuel, is specified, the numbers in the third column are the same as those in the third column of Table A.

TABLE A.

VARIOUS ENGINES IN WHICH THE TEMPERATURE OF THE BOILER IS 140° C. AND THAT OF THE CONDENSER 30° C.

Theoretical Duty for each Unit of Heat transmitted, 440* *foot-pounds.*

CASES.	Work produced for each lb. of coal consumed.	Work produced for each lb. of watereva-porated.	Work produced for each unit of heat transmitted.	Percentage of theoretical duty.
	Ft.-lbs.	Ft.-lbs.	Ft.-lbs.	
(1) Fowey Consols experiment, reported in 1845	1,330,734	156,556	253	57.5
(2) Taylor's engine at the United Mines, working in 1840	1,042,553	122,653	198.4	45.1
(3) French engines, according to contract.......	....	98,427	159	36.1
(4) English engines, according to contract..	565,700	80,814	130.8	29.7
(5) Average actual performance of Cornish engines...	585,106	68,836	111.3	25.3
(6) Common engines, consuming 12 lbs. of best coal per horse-power per hour....	165,000	23,571	38.1	8.6
(7) Improved engines with expansion cylinders, consuming an equivalent to 4 lbs. of best coal per horse-power per hour..............	495,000	70,710	114.4	26

* [Note added March 15, 1881. Total work for thermal unit, 1390 (Joule), 377.1 corrected by the dynamical theory, March 15, 1851.

$$377.1 = .2713 \times 1390,$$

$$253 = .1820 \times 1390 = \frac{1}{5.49} \times 1390.]$$

TABLE B.

VARIOUS ENGINES IN WHICH THE TEMPERATURE OF THE BOILER IS 121° C.* AND THAT OF THE CONDENSER 30° C.

Theoretical Duty for each Unit of Heat transmitted, 371 *foot-pounds.*

CASES.	Work produced for each lb. of coal consumed.	Work produced for each lb. of water evaporated.	Work produced for each unit of heat transmitted.	Percentage of theoretical duty.
	Ft.-lbs.	Ft.-lbs.	Ft.-lbs.	
(3) French engines, according to contract.......	....	98,427	160.3	43.2
(4) English engines, according to contract..	565,700	$\frac{614}{618} \times 80{,}814$	130.8	35
(6) Common engines, consuming 12 lbs. of coal per horse-power per hour	165,000	$\frac{614}{618} \times 23{,}571$	38.1	10.3
(7) Improved engines with expansion cylinders, consuming an equivalent to 4 lbs. best coal per horse-power per hour	495,000	$\frac{614}{618} \times 70{,}710$	114.4	30.7

* Pressure 15 lbs. on the square inch.

APPENDIX A.

EXTRACTS FROM UNPUBLISHED WRITINGS OF CARNOT.

I. Notes.

Let us first open at the memoranda relating to his daily occupations:

"Plan in the morning the work of the day, and reflect in the evening on what has been done."

"Carry when walking a book, and a note-book to preserve the ideas, and a piece of bread in order to prolong the walk if need be."

"Vary the mental and bodily exercises with dancing, horsemanship, swimming, fencing with sword and with sabre, shooting with gun and pistol, skating, the sling, stilts, tennis, bowls; hop on one foot, cross the arms, jump high and far, turn on one foot propped against the wall, exercise in shirt in the evening to get up a perspiration before going to bed; turning, joinery, gardening, reading while walking, declamation, singing, violin, versification, musical composition; eight hours of sleep; a walk on awakening, before and after eating; great so-

briety; eat slowly, little, and often; avoid idleness and useless meditation."

Then come more general precepts:

"Adopt good habits when I change my method of life."

"Never turn to the past unless to enlighten the future. Regrets are useless."

"Form resolutions in advance in order not to reflect during action. Then obey thyself blindly."

"The promptitude of resolutions most frequently accords with their justice."

"Yield frequently to the first inspiration. Too much meditation on the same subject ends by suggesting the worst part, or at least causes loss of precious time."

"Suffer slight disagreeables without seeming to perceive them, but repulse decisively any one who evidently intends to injure or humiliate you."

"One should never feign a character that he has not, or affect a character that he cannot sustain."

"Self-possession without self-sufficiency. Courage without effrontery."

"Make intimate acquaintances only with much circumspection; perfect confidence in those who

have been thoroughly tested. Nothing to do with others."

"Question thyself to learn what will please others."

"No useless discourse. All conversation which does not serve to enlighten ourselves or others, to interest the heart or amuse the mind, is hurtful."

"Speak little of what you know, and not at all of what you do not know."

"Why not say more frequently, 'I do not know'?"

"Speak to every one of that which he knows best. This will put him at his ease, and be profitable to you."

"Abstain from all pleasantry which could wound."

"Employ only expressions of the most perfect propriety."

"Listen attentively to your interlocutor, and so prepare him to listen in the same way to your reply, and predispose him in favor of your arguments."

"Show neither passion nor weariness in discussion."

"Never direct an argument against any one. If you know some particulars against your adversary, you have a right to make him aware of it to keep

him under control, but proceed with discretion, and do not wound him before others."

"When discussion degenerates into dispute, be silent; this is not to declare yourself beaten."

"How much modesty adds to merit! A man of talent who conceals his knowledge is like a branch bending under a weight of fruit."

"Why try to be witty? I would rather be thought stupid and modest than witty and pretentious."

"Men desire nothing so much as to make themselves envied."

"Egotism is the most common and most hated of all vices. Properly speaking, it is the only one which should be hated."

"The pleasures of self-love are the only ones that can really be turned into ridicule."

"I do not know why these two expressions, good sense and common sense, are confounded. There is nothing less common than good sense."

"The strain of suffering causes the mind to decay."

We will quote one of those misanthropic sallies the rarity of which we are glad to remark:

"It must be that all honest people are in the galleys; only knaves are to be met with elsewhere."

But serenity of mind returns immediately after the above:

"I rejoice for all the misfortunes which might have happened to me, and which I have escaped."

"Life is a short enough passage. I am half the journey. I will complete the remainder as I can."

"Hope being the greatest of all blessings, it is necessary, in order to be happy, to sacrifice the present to the future."

"Let us not be exacting; perfection is so rare."

"Indulgence! Indulgence!"

"The more nearly an object approaches perfection, the more we notice its slightest defects."

"To neglect the opportunity of an innocent pleasure is a loss to ourselves. It is to act like a spendthrift."

"*Recherché* pleasures cause simple pleasures to lose all their attractions."

"It may sometimes be necessary to yield the right, but how is one to recover it when wanted?"

"Love is almost the only passion that the good man may avow. It is the only one which accords with delicacy."

"Do nothing that all the world may not know."

"The truly wise man is he who loves virtue for its own sake."

"We say that man is an egotist, and nevertheless his sweetest pleasures come to him through others. He only tastes them on condition of sharing them."

"If one could continually satisfy his desires, he would never have time to desire. Happiness then is necessarily composed of alternatives. It could not exist at a constant level."

On the subject of nations and conquerors:

"To each conqueror can be said, when he has ceased tormenting our poor globe, 'Would you not have been able to tilt equally well against a little globe of pasteboard?'"

"The laws of war, do they say? As if war were not the destruction of all laws."

"War has been represented as necessary to prevent the too rapid increase of the population, but war mows down the flower of the young men, while it spares the men disgraced by nature. Hence it tends to the degeneration of the species."

Then the writer turns his shafts against medicine:

"In some respects medicine is directly opposed to the will of nature, which tends to perpetuate the strongest and best of the species, and to abandon

the delicate to a thousand forms of destruction. This is what occurs among animals and savage men. Only the most robust attain the adult age, and these only reproduce the species. Medicine and the aids of the social state prolong the lives of feeble creatures whose posterity is usually equally feeble. Among the Spartans, barbarous regulations put an end to the existence of mal-formed infants, that the strength and beauty of the race might be preserved. Such regulations are antipathetic to our customs; nevertheless it might be desirable that we should devote ourselves to the preservation of the human race from the causes of weakness and degeneracy."

"The decadence of the Greeks and Romans without change of race proves the influence of institutions upon customs."

We will give here a fragment on political economy, to show the variety contained in the pages on which we draw:

"According to the system of modern economists, it would be desirable that the government should interfere as little as possible in the commerce and industry of the country. Nevertheless we cannot deny that in certain circumstances this intervention is very useful."

"Taxes are regarded by economists as an evil, but as a necessary evil, since they provide for public expenses. Consequently, economists think that if the government possessed sufficient revenues, in domains for example, the suppression of all taxes would be a desirable measure."

"Taxes are a means of influencing production and commerce to give to them a direction which they would not naturally have taken. Such an influence may undoubtedly have disagreeable consequences if the taxes are imposed without discrimination or exclusively for a fiscal purpose, but it is entirely otherwise if wisdom and tact preside at their institution."

"A tax on the rent of a farm would be much better than a tax on the land itself. Proprietors then could only avoid taxes by themselves improving their property. As it is, they merely collect the rents, and usually employ their surplus in unproductive expenditure, while the proprietary farmers voluntarily devote theirs to the improvement of the land."

"A tax on the farms would then result in the proprietors themselves working the lands, and this would mean better cultivation, and improvements which would yield returns indeed, but at too remote a period for the tenant. It would tend to a

division of landed property, men of small fortune uniting in the purchase with capitalists who seek only the rent or payment for the land."

"Great capitalists could not themselves cultivate vast extents of land, and not wanting to diminish their revenues by renting them, would be induced to sell portions suitable for cultivation by their new owners, and would then carry their money into new industrial and commercial enterprises."

"The competition of the sellers would cause a momentary fall in the price of the lands, and would enable small farmers to become land-owners. The number of vast estates often badly managed would then be diminished, and considerable fortunes, changing hands more easily, would naturally pass into those which would be most likely to increase their value."

"Proprietors, becoming cultivators to escape the taxes, would settle in the country, where their presence would disseminate intelligence and comfort; their revenues, before spent unprofitably, would then pay expenses and improvements on their property."

"The establishment of such a tax would certainly find many opponents among proprietors, landed non-cultivators who form in fact the influ-

ential *personnel* in the state, for it is they who usually make the laws."

"Perhaps it would be necessary to weaken their opposition by not subjecting the actual proprietors to the new tax, which might take effect only with the next change either by sale or by inheritance. A restriction of the right of transfer would also facilitate the passage from one situation to the other. All changes in taxes should, as a general thing, be made gradually, in order to avoid sudden changes of fortune."

"We may consider the renting of a property for several years as a sale of the usufruct during the time of the lease. Now nine years' possession, for example, is equal to more than a third of the value of the property, supposing the annual product to be one twentieth of the capital. It would then be reasonable to apply to this sort of sale the laws which govern that of landed property, and consequently the mutation tax. The person who cannot or will not cultivate his soil, instead of alienating the property itself, binds himself to alienate the usufruct for a time, and the price is paid at stated intervals instead of all at once. There is farm rent."

"Now it is by a fiction that the purchaser pays the mutation tax. In fact, it is always the seller

who pays it. The buyer compares the money that he spends with the advantage that he gains, and this comparison determines it. If he did not make money out of it he would not buy it. When the registration tax did not exist, the purchaser had to pay the same sum for the same purpose, and this sum went into the pocket of the seller."

"Proprietors of lands, then, after all, have to bear the mutation taxes. All increase of these taxes is a loss for them, and these taxes are heavier on the small proprietors than on the large, because their changes are more frequent. The tax on the farms, on the contrary, would bear more heavily on large estates."

"The tax on farms not affecting the owners of timber, would be made up by a tax on the felling, a very justifiable tax, for standing timber is landed property. Standing timber is often worth much more than the land on which it stands."

Finally, we will give some thoughts which reveal the religious sentiments of Sadi Carnot:

"Men attribute to chance those events of the causes of which they are ignorant. If they succeed in divining these causes, chance disappears. To say that a thing has happened by chance,

is to say that we have not been able to foresee it. I do not myself believe that any other acceptation can be given to this word. What to an ignorant man is chance, cannot be chance to one better instructed."

"If human reason is incapable of discovering the mysteries of Divinity, why has not Divinity made human reason more clear-sighted?"

"God cannot punish man for not believing when he could so easily have enlightened and convinced him."

"If God is absolutely good, why should He punish the sinner for all eternity, since He does not lead him to good, or give him an example?"

"According to the doctrine of the church, God resembles a sphinx proposing enigmas, and devouring those who cannot guess them."

"The church attributes to God all human passions—anger, desire for vengeance, curiosity, tyranny, partiality, idleness."

"If Christianity were pruned of all which is not Christ, this religion would be the simplest in the world."

"What motives have influenced the writers who have rejected all religious systems? Is it the conviction that the ideas which they oppose are all

injurious to society? Have they not rather included in the same proscription religion and the abuse of it?"

"The belief in an *all-powerful* Being, who loves us and watches over us, gives to the mind great strength to endure misfortune."

"A religion suited to the soul and preached by men worthy of respect would exercise the most salutary influence upon society and customs."

II. Notes of Sadi Carnot on Mathematics, Physics, and other Subjects.

Up to the present time the changes caused in the temperature of bodies by motion have been very little studied. This class of phenomena merits, however, the attention of observers. When bodies are in motion, especially when that motion disappears, or when it produces motive power, remarkable changes take place in the distribution of heat, and perhaps in its quantity.

We will collect a few facts which exhibit this phenomenon most clearly.

1. *The Collision of Bodies.*—We know that in the collision of bodies there is always expenditure of motive power. Perfectly elastic bodies only form an exception, and none such are found in nature.

We also know that always in the collision of bodies there occurs a change of temperature, an elevation of temperature. We cannot, as did M. Berthollet, attribute the heat set free in this case to the reduction of the volume of the body; for when this reduction has reached its limit the liberation of heat would cease. Now this does not occur.

It is sufficient that the body change form by percussion, without change of volume, to produce disengagement of heat.

If, for example, we take a cube of lead and strike it successively on each of its faces, there will always be heat liberated, without sensible diminution in this disengagement, so long as the blows are continued with equal force. This does not occur when medals are struck. In this case the metal cannot change form after the first blows of the die, and the effect of the collision is not conveyed to the medal, but to the threads of the screw which are strained, and to its supports.

It would seem, then, that heat set free should be attributed to the friction of the molecules of the metal, which change place relatively to each other, that is, the heat is set free just where the moving-force is expended.

A similar remark will apply in regard to the col-

lision of two bodies of differing hardness—lead and iron for instance. The first of these metals becomes very hot, while the second does not vary sensibly in temperature. But the motive power is almost wholly exhausted in changing the form of the first of these metals. We may also cite, as a fact of the same nature, the heat produced by the extension of a metallic rod just before it breaks. Experiment has proved that, other things being equal, the greater the elongation before rupture, the more considerable is the elevation of temperature.

(2) [The remainder is blank.]

When a hypothesis no longer suffices to explain phenomena, it should be abandoned.

This is the case with the hypothesis which regards caloric as matter, as a subtile fluid.

The experimental facts tending to destroy this theory are as follows:

(1) The development of heat by percussion or the friction of bodies (experiments of Rumford, friction of wheels on their spindles, on the axles, experiments to be made). Here the elevation of temperature takes place at the same time in the body rubbing and the body rubbed. Moreover, they do not change perceptibly in form or nature

(to be proved). Thus heat is produced by motion. If it is matter, it must be admitted that the matter is created by motion.

(2) When an air-pump is worked, and at the same time air is admitted into the receiver, the temperature remains constant in the receiver. It remains constant on the outside. Consequently, the air compressed by the pumps must rise in temperature above the air outside, and it is expelled at a higher temperature. The air enters then at a temperature of 10°, for instance, and leaves at another, 10° + 90° or 100°, for example. Thus heat has been created by motion.

(3) If the air in a reservoir is compressed, and at the same time allowed to escape through a little opening, there is by the compression elevation of temperature, by the escape lowering of temperature (according to Gay-Lussac and Welter). The air then enters at one side at one temperature and escapes at the other side at a higher temperature, from which follows the same conclusion as in the preceding case.

(Experiment to be made: To fit to a high-pressure boiler a cock and a tube leading to it and emptying into the atmosphere; to open the cock a little way, and present a thermometer to the outlet of the steam; to see if it remains at 100° or more;

to see if steam is liquefied in the pipe; to see whether it comes out cloudy or transparent.)

(4) The elevation of temperature which takes place at the time of the entrance of the air into the vacuum, an elevation that cannot be attributed to the compression of the air remaining (air which may be replaced by steam), can therefore be attributed only to the friction of the air against the walls of the opening, or against the interior of the receiver, or against itself.

(5) M. Gay-Lussac showed (it is said) that if two receivers were put in communication with each other, the one a vacuum, the other full of air, the temperature would rise in one as much as it would fall in the other. If, then, both be compressed one half, the first would return to its previous temperature and the second to a much higher one. Mixing them, the whole mass would be heated.

When the air enters a vacuum, its passage through one small opening and the motion imparted to it in the interior appear to produce elevation of temperature.

We may be allowed to express here an hypothesis in regard to the nature of heat.

At present, light is generally regarded as the

result of a vibratory movement of the ethereal fluid. Light produces heat, or at least accompanies the radiating heat, and moves with the same velocity as heat. Radiating heat is then a vibratory movement. It would be ridiculous to suppose that it is an emission of matter while the light which accompanies it could be only a movement.

Could a motion (that of radiating heat) produce matter (caloric)?

No, undoubtedly; it can only produce a motion. Heat is then the result of a motion.

Then it is plain that it could be produced by the consumption of motive power, and that it could produce this power.

All the other phenomena—composition and decomposition of bodies, passage to the gaseous state, specific heat, equilibrium of heat, its more or less easy transmission, its constancy in experiments with the calorimeter—could be explained by this hypothesis. But it would be difficult to explain why, in the development of motive power by heat, a cold body is necessary; why, in consuming the heat of a warm body, motion cannot be produced.

It appears very difficult to penetrate into the real essence of bodies. To avoid erroneous reasoning, it would be necessary to investigate carefully

the source of our knowledge in regard to the nature of bodies, their form, their forces; to see what the primitive notions are, to see from what impressions they are derived; to see how one is raised successively to the different degrees of abstraction.

Is heat the result of a vibratory motion of molecules? If this is so, quantity of heat is simply quantity of motive power. As long as motive power is employed to produce vibratory movements, the quantity of heat must be unchangeable; which seems to follow from experiments with the calorimeter; but when it passes into movements of sensible extent, the quantity of heat can no longer remain constant.

Can examples be found of the production of motive power with actual consumption of heat? It seems that we may find production of heat with consumption of motive power (re-entrance of the air into a vacuum, for example).

What is the cause of the production of heat in combinations of substances? What is radiant caloric?

Liquefaction of bodies, solidification of liquids,

crystallization—are they not forms of combinations of integrant molecules?

Supposing heat due to a vibratory movement, how can the passage from the solid or the liquid to the gaseous state be explained?

When motive power is produced by the passage of heat from the body A to the body B, is the quantity of this heat which arrives at B (if it is not the same as that which has been taken from A, if a portion has really been consumed to produce motive power) the same whatever may be the substance employed to realize the motive power?

Is there any way of using more heat in the production of motive power, and of causing less to reach the body B? Could we even utilize it entirely, allowing none to go to the body B? If this were possible, motive power could be created without consumption of combustible, and by mere destruction of the heat of bodies.

Is it absolutely certain that steam after having operated an engine and produced motive power can raise the temperature of the water of condensation as if it had been conducted directly into it?

Reasoning shows us that there cannot be loss of

living force, or, which is the same thing, of motive power, if the bodies act upon each other without directly touching each other, without actual collision. Now everything leads us to believe that the molecules of bodies are always separated from each other by some space, that they are never actually in contact. If they touched each other, they would remain united, and consequently change form.

If the molecules of bodies are never in close contact with each other whatever may be the forces which separate or attract them, there can never be either production or loss of motive power in nature. This power must be as unchangeable in quantity as matter. Then the direct re-establishment of equilibrium of the caloric, and its re-establishment with production of motive power, would be essentially different from each other.

Heat is simply motive power, or rather motion which has changed form. It is a movement among the particles of bodies. Wherever there is destruction of motive power there is, at the same time, production of heat in quantity exactly proportional to the quantity of motive power destroyed. Reciprocally, wherever there is destruction of heat, there is production of motive power.

We can then establish the general proposition that motive power is, in quantity, invariable in nature; that it is, correctly speaking, never either produced or destroyed. It is true that it changes form, that is, it produces sometimes one sort of motion, sometimes another, but it is never annihilated.

According to some ideas that I have formed on the theory of heat, the production of a unit of motive power necessitates the destruction of 2.70 units of heats.

A machine which would produce 20 units of motive power per kilogram of coal ought to destroy $\frac{20 \times 2.70}{7000}$ of the heat developed by the combustion.

$\frac{20 \times 2.70}{7000} = \frac{8}{1000}$ about; that is, less than $\frac{1}{100}$.

(Each unit of motive power, or dyname, representing the weight of one cubic metre of water raised to the height of one metre.)

Experiments to be made on Heat and Motive Power.

To repeat Rumford's experiments in the drilling of a metal in water, but to measure the motive power consumed at the same time as the heat pro-

duced; same experiments on several metals and on wood.

To strike a piece of lead in various ways, to measure the motive power consumed and the heat produced. Same experiments on other metals.

To strongly agitate water in a small cask or in a double-acting pump having a piston pierced with a small opening.

Experiment of the same sort on the agitation of mercury, alcohol, air and other gases. To measure the motive power consumed and heat produced.

To admit air into a vacuum or into air more or less rarefied; *id.* for other gases or vapors. To examine the elevation of temperature by means of the manometer and the thermometer of Bréguet. Estimation of the error of the thermometer in the time required for the air to vary a certain number of degrees. These experiments would serve to measure the changes which take place in the temperature of the gas during its changes of volume. They would also furnish means of comparing these changes with the quantities of motive power produced or consumed.

Expel the air from a large reservoir in which it is compressed, and check its velocity in a large pipe in

which solid bodies have been placed; measure the temperature when it has become uniform. See if it is the same as in the reservoir. Same experiments with other gases and with vapor formed under different pressures.

To repeat Dalton's experiments and carry them on to pressures of thirty or forty atmospheres. To measure the constituent heat of the vapor within these limits.

Id. on the vapor of alcohol, of ether, of essence of turpentine, of mercury, to prove whether the agent employed makes any difference in the production of motive power.

Id. on water charged with a deliquescent salt, the calcium chloride, for instance.

Is the law of tensions always the same? To measure the specific heat of vapor.

Experiments to be made on the Tension of Vapors.

A graduated capillary tube filled with water, mercury, or with oil and air. Plunge this tube into a bath of oil, of mercury, or of melted lead. To measure the temperature by an air thermometer.

Same experiments with alcohol, ether, sulphide of carbon, muriatic ether, essence of turpentine, sulphur, phosphorus.

Experiments on the tension of steam with a boiler, and a thermometric tube full of air. A thermometer will be placed in a tube immersed in the boiler, open outwards and filled with oil or mercury.

Experiments by means of a simple capillary tube filled with three successive parts—first of air, second of mercury, third of water or other liquid of which the tension can be measured (of alcohol, of ether, of essence of turpentine, of lavender, of sulphide of carbon, of muriatic ether, etc.). One end of the tube may be immersed in a bath of mercury or oil, the temperature of which is to be measured. The column of mercury can be made long enough to allow of the air being previously compressed or rarefied.

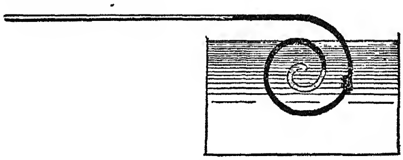

FIG. 6.

The tube will be bent into a spiral at one end, the straight part being graduated (thus permitting the tension of mercurial vapor to be measured).

Experiments on the tension of vapors at low

temperature, with a thermometric tube bent round, and filled partly with mercury, partly with water or alcohol. The mercury will operate by its weight. The upper part of the tube will be empty and sealed, or fully open to the atmosphere.


Fig. 7.

The bulb will be immersed in water the temperature of which is to be measured. If the tube is sealed, the upper part must be cooled.

The bulb might contain water, ether, or essence of turpentine.

If the tube is sealed, the tension of mercurial vapor could be measured.

Experiments on the constituent heat of vapors by means of a barometric tube having two enlarged bulbs. One of the bulbs may be immersed in cold water, and the elevation of temperature of this water will indicate the constituent heat of the vapor.

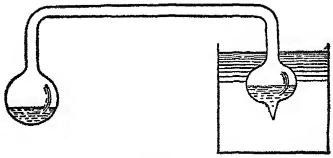
Fig. 8.

The other bulb may be warmed either by boiling liquid or by fire.

Water, alcohol, steam, ether, mercury, acetic acid, sulphide of carbon.

The operation may be repeated and add the results.

Experiments to be made on Gases and Vapors.

To measure the temperature acquired by the air introduced into a vacuum or space containing previously rarefied air.

If the vacuum is made under the glass receiver of an air-pump, and the cock admitting the outer air be suddenly opened, the introduction of this air will cause a Bréguet thermometer to rise to 50° or 60°. To examine the movement of this thermometer when the reintroduction takes place only by degrees, to compare it with the movement of the manometer.

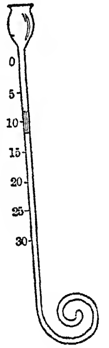

FIG. 9.

Construction of a manometer which may give the pressure almost instantaneously.

Imagine a capillary tube bent into a spiral at one end, and having one extremity closed, the other open. This tube will be perfectly dry and a small index of mercury may be introduced into it. The diameter of the tube will be small

enough for the air enclosed in it to take almost instantly the temperature of the glass. We shall try to ascertain the time necessary for the establishment of this equilibrium of temperature by placing the tube under the receiver of the air-pump, making a partial vacuum, and admitting the air. We shall see whether, some seconds after the introduction, the index perceptibly moves. The index must be of very light weight to avoid oscillation as much as possible.

For the same reason, the capillary tube should be also as narrow as possible. If the straight part of the tube is equal to the bent part and the index be placed at the beginning of the bent part, for a pressure equal to atmospheric pressure, it would not be necessary to subject the instrument to a less pressure than $\frac{1}{2}$ atmosphere. It is between these two limits that it would serve as a measure.

It might end in an open enlargement to prevent the projection of the mercury outside the tube. Disposed in this way, it could be used as a general measure for pressures between p and $\frac{1}{2}p$; p being anything whatever. The apparatus will be fastened to a board bearing a graduated scale placed against the straight tube. The scale will be, for instance, numbered by fives or tens. A corresponding table denoting pressures would be required.

Placing the instrument under the receiver and forming a partial vacuum, the index will rise into the enlargement. Then, admitting the air by degrees and very slowly, we may note the correspondence between the heights of the ordinary mercury manometer and the point which will be reached by the lower face of the index of the instrument. This will answer to form a comparative table of the pressures and the numbers of the scale. The pressures would be represented by their relations to the observed pressure at the moment of the passage of the index over zero, for any other fixed number of the scale.

Thus, for example, suppose that we observed on the manometer 400 or n millimetres of mercury when the index is on o, then n' when the index is on 1, n'' when on 2, and so on. This will give the ratios $\frac{n'}{n}, \frac{n''}{n}, \ldots$ which must be inscribed in the table. Then n could be varied at pleasure, and the table could still be used.

In fact, according to the law of Mariotte, volumes preserving the same ratios, pressures should also preserve the same ratios to each other.

Let p be the pressure when the index is on o, v the volume of air at the same moment, p' and v' the same pressures and volume at the moment

when the index is on 1. Whether the air be expelled or admitted the pressures would be instead of p and p', q and q'. But there would follow

$$p : p' :: v' : v \qquad \text{and} \qquad q : q' :: v' : v;$$

then $$p : p' :: q : q'.$$

We should moreover work at a uniform temperature and note the variations.

If the straight part of the tube were perfectly calibrated, the volumes, and consequently the pressures, would form a geometrical progression, when the figures of the scale would be found to be in arithmetical progression, and a table of logarithms would enable one to be found from the other.

In order to increase as required the mass of air enclosed in the tube the instrument must be placed on its side or flat, in the air-pump receivers. The mercury index would be placed in the lateral part of the enlargement of the tube, and the atmospheric air would enter. The instrument might also be heated in this position.

Care must be taken to admit only very dry air, which could be obtained by placing under the receiver calcium chloride or any other substance which absorbs moisture greedily.

Instead of bending the tube into a spiral, it might be bent in the middle in the form of a U, or it might be better to form three, four or more

parallel branches. Making the tube very long, the index would have a larger range for the same changes of pressure, and the results produced could then be measured by a slight variation in density in the air of the receiver.

Comparison of the Rapidity with which the Air cools in the Receiver and in the Tube.

Let us suppose, what I believe to be very near the truth, that the heat absorbed is proportional to the surface of the bodies in contact. From this we can infer without difficulty, that the rapidity of the cooling of the air in two cylindrical tubes would be inversely as their diameters.

If the receiver is considered as a tube of two decimetres in diameter, and the manometer as a tube of one millimetre diameter, the rapidity of the cooling of the air would be in the ratio, very nearly, of 1 to 200.

Extent of the Movement of the Index.

Suppose the tube turned up on itself five times and having a total length of 1 metre; a variation of density equal to $\frac{1}{10}$ in the air will give a movement of 1 decimetre; a variation of heat of 1 degree supposed to be equivalent to a variation of density of $\frac{1}{266}$ will give $\frac{1}{266}$ of a metre, or about

$3^{mm}.70$, quite an appreciable quantity. As to the time required to move the mercury index, regard being had to its mass, if we suppose it 1 centimetre long, and the variation of pressure $\frac{1}{100}$ of an atmosphere, it would require about $\frac{1}{6}$ of a second to make it pass over one decimetre.

Use of the Instrument in Measuring the Variations of the Tensions of the Air under a Pneumatic Receiver.

At each stroke of the piston which expands the air under the pneumatic receiver when a vacuum is to be created, a lowering of pressure is produced, and undoubtedly a change of temperature. It can be determined approximately, at least, by observing the position of the manometer at the instant after the dilatation has taken place, and again after a time long enough for the temperature to have returned to its original point, that of the surrounding bodies. Comparison of the elastic force in the two cases will lead to comparison of the temperatures.

The temperature having returned to its original point, we will give a second stroke of the piston which will rarefy the air more than the former, and thus we will make two observations of the manometer, before and after the return to the former temperature. And so on.

APPENDIX B.

CARNOT'S FOOT-NOTES.

NOTE A.—The objection may perhaps be raised here, that perpetual motion, demonstrated to be impossible by mechanical action alone, may possibly not be so if the power either of heat or electricity be exerted; but is it possible to conceive the phenomena of heat and electricity as due to anything else than some kind of motion of the body, and as such should they not be subjected to the general laws of mechanics? Do we not know besides, *à posteriori*, that all the attempts made to produce perpetual motion by any means whatever have been fruitless?—that we have never succeeded in producing a motion veritably perpetual, that is, a motion which will continue forever without alteration in the bodies set to work to accomplish it? The electromotor apparatus (the pile of Volta) has sometimes been regarded as capable of producing perpetual motion; attempts have been made to realize this idea by constructing dry piles said to be unchangeable; but however it has been done, the apparatus has always exhibited sensible

deteriorations when its action has been sustained for a time with any energy.

The general and philosophic acceptation of the words *perpetual motion* should include not only a motion susceptible of indefinitely continuing itself after a first impulse received, but the action of an apparatus, of any construction whatever, capable of creating motive power in unlimited quantity, capable of starting from rest all the bodies of nature if they should be found in that condition, of overcoming their inertia; capable, finally, of finding in itself the forces necessary to move the whole universe, to prolong, to accelerate incessantly, its motion. Such would be a veritable creation of motive power. If this were a possibility, it would be useless to seek in currents of air and water or in combustibles this motive power. We should have at our disposal an inexhaustible source upon which we could draw at will.

NOTE B.—The experimental facts which best prove the change of temperature of gases by compression or dilatation are the following:

(1) The fall of the thermometer placed under the receiver of a pneumatic machine in which a vacuum has been produced. This fall is very sensible on the Bréguet thermometer: it may exceed 40° or 50°. The mist which forms in this case

seems to be due to the condensation of the watery vapor caused by the cooling of the air.

(2) The inflammation of German tinder in the so-called pneumatic tinder-boxes; which are, as we know, little pump-chambers in which the air is rapidly compressed.

(3) The fall of a thermometer placed in a space where the air has been first compressed and then allowed to escape by the opening of a cock.

(4) The results of experiments on the velocity of sound: M. de Laplace has shown that, in order to secure results accurately by theory and computation, it is necessary to assume the heating of the air by sudden compression.

The only fact which may be adduced in opposition to the above is an experiment of MM. Gay-Lussac and Welter, described in the *Annales de Chimie et de Physique.* A small opening having been made in a large reservoir of compressed air, and the ball of a thermometer having been introduced into the current of air which passes out through this opening, no sensible fall of the temperature denoted by the thermometer has been observed.

Two explanations of this fact may be given: (1) The striking of the air against the walls of the opening by which it escapes may develop heat in

observable quantity. (2) The air which has just touched the bowl of the thermometer possibly takes again by its collision with this bowl, or rather by the effect of the *détour* which it is forced to make by its rencounter, a density equal to that which it had in the receiver,—much as the water of a current rises against a fixed obstacle, above its level.

The change of temperature occasioned in the gas by the change of volume may be regarded as one of the most important facts of Physics, because of the numerous consequences which it entails, and at the same time as one of the most difficult to illustrate, and to measure by decisive experiments. It seems to present in some respects singular anomalies.

Is it not to the cooling of the air by dilatation that the cold of the higher regions of the atmosphere must be attributed? The reasons given heretofore as an explanation of this cold are entirely insufficient; it has been said that the air of the elevated regions receiving little reflected heat from the earth, and radiating towards celestial space, would lose caloric, and that this is the cause of its cooling; but this explanation is refuted by the fact that, at an equal height, cold reigns with equal and even more intensity on the elevated

plains than on the summit of the mountains, or in those portions of the atmosphere distant from the sun.

NOTE C.—We see no reason for admitting, *à priori,* the constancy of the specific heat of bodies at different temperatures, that is, to admit that equal quantities of heat will produce equal increments of temperature, when this body changes neither its state nor its density; when, for example, it is an elastic fluid enclosed in a fixed space. Direct experiments on solid and liquid bodies have proved that between zero and 100°, equal increments in the quantities of heat would produce nearly equal increments of temperature. But the more recent experiments of MM. Dulong and Petit (see *Annales de Chimie et de Physique,* February, March, and April, 1818) have shown that this correspondence no longer continues at temperatures much above 100°, whether these temperatures be measured on the mercury thermometer or on the air thermometer.

Not only do the specific heats not remain the same at different temperatures, but, also, they do not preserve the same ratios among themselves, so that no thermometric scale could establish the constancy of all the specific heats. It would have been interesting to prove whether the same irregulari-

ties exist for gaseous substances, but such experiments presented almost insurmountable difficulties.

The irregularities of specific heats of solid bodies might have been attributed, it would seem, to the latent heat employed to produce a beginning of fusion—a softening which occurs in most bodies long before complete fusion. We might support this opinion by the following statement: According to the experiments of MM. Dulong and Petit, the increase of specific heat with the temperature is more rapid in solids than in liquids, although the latter possess considerably more dilatability. The cause of irregularity just referred to, if it is real, would disappear entirely in gases.

Note D.—In order to determine the arbitrary constants A, B, A', B', in accordance with the results in M. Dalton's table, we must begin by computing the volume of the vapor as determined by its pressure and temperature,—a result which is easily accomplished by reference to the laws of Mariotte and Gay-Lussac, the weight of the vapor being fixed.

The volume will be given by the equation

$$v = c\,\frac{267 + t}{p},$$

in which v is this volume, t the temperature, p the

pressure, and c a constant quantity depending on the weight of the vapor and on the units chosen. We give here the table of the volumes occupied by a gramme of vapor formed at different temperatures, and consequently under different pressures.

t or degrees Centigrade.	p or tension of the vapor expressed in millimetres of mercury.	v or volume of a gramme of vapor expressed in litres.
°	mm.	lit.
0	5.060	185.0
20	17.32	58.2
40	53.00	20.4
60	144.6	7.96
80	352.1	3.47
100	760.0	1.70

The first two columns of this table are taken from the *Traité de Physique* of M. Biot (vol. i., p. 272 and 531). The third is calculated by means of the above formula, and in accordance with the result of experiment, indicating that water vaporized under atmospheric pressure occupies a space 1700 times as great as in the liquid state.

By using three numbers of the first column and three corresponding numbers of the third column, we can easily determine the constants of our equation

$$t = \frac{A + B \log v}{A' + B' \log v}.$$

We will not enter into the details of the calculation necessary to determine these quantities. It is sufficient to say that the following values,

$$A = 2268, \qquad A' = 19.64,$$
$$B = -1000, \qquad B' = \ \ 3.30,$$

satisfy fairly well the prescribed conditions, so that the equation

$$t = \frac{2268 - 1000 \log v}{19.64 + 3.30 \log v}$$

expresses very nearly the relation which exists between the volume of the vapor and its temperature. We may remark here that the quantity B' is positive and very small, which tends to confirm this proposition—that the specific heat of an elastic fluid increases with the volume, but follows a slow progression.

NOTE E.—Were we to admit the constancy of the specific heat of a gas when its volume does not change, but when its temperature varies, analysis would show a relation between the motive power and the thermometric degree. We will show how this is, and this will also give us occasion to show how some of the propositions established above should be expressed in algebraic language.

Let r be the quantity of motive power produced by the expansion of a given quantity of air passing

from the volume of one litre to the volume of v litres under constant temperature. If v increases by the infinitely small quantity dv, r will increase by the quantity dr, which, according to the nature of motive power, will be equal to the increase dv of volume multiplied by the expansive force which the elastic fluid then possesses; let p be this expansive force. We should have the equation

$$dr = pdv. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let us suppose the constant temperature under which the dilatation takes place equal to t degrees Centigrade. If we call q the elastic force of the air occupying the volume 1 litre at the same temperature t, we shall have, according to the law of Mariotte,

$$\frac{v}{1} = \frac{q}{p}, \text{ whence } p = \frac{q}{v}.$$

If now P is the elastic force of this same air at the constant volume 1, but at the temperature zero, we shall have, according to the rule of M. Gay-Lussac,

$$q = P + P\frac{t}{267} = \frac{P}{267}(267 + t);$$

whence

$$q = p = \frac{P}{267}\frac{267 + t}{v}.$$

If, to abridge, we call N the quantity $\frac{P}{726}$, the equation would become

$$p = N\frac{t + 267}{v};$$

whence we deduce, according to equation (1),

$$dr = N\frac{t + 267}{v}dv.$$

Regarding t as constant, and taking the integral of the two numbers, we shall have

$$r = N(t + 267)\log v + C.$$

If we suppose $r = 0$ when $v = 1$, we shall have $C = 0$; whence

$$r = N(t + 267)\log v. \quad . \quad . \quad . \quad (2)$$

This is the motive power produced by the expansion of the air which, under the temperature t, has passed from the volume 1 to the volume v. If instead of working at the temperature t we work in precisely the same manner at the temperature $t + dt$, the power developed will be

$$r + \delta r = N(t + dt + 267)\log v.$$

Subtracting equation (2), we have

$$\delta r = N\log v dt. \quad . \quad . \quad . \quad . \quad (3)$$

Let e be the quantity of heat employed to maintain the temperature of the gas constant during its

dilatation. According to the reasoning of page 69, δr will be the power developed by the fall of the quantity e of heat from the degree $t + td$ to the degree t. If we call u the motive power developed by the fall of unity of heat from the degree t to the degree zero, as, according to the general principle established page 68, this quantity u ought to depend solely on t, it could be represented by the function Ft, whence $u = Ft$.

When t is increased it becomes $t + td$, u becomes $u + du$; whence

$$u + du = F(t + dt).$$

Subtracting the preceding equation, we have

$$du = F(t + dt) - Ft = F'tdt.$$

This is evidently the quantity of motive power produced by the fall of unity of heat from the temperature $t + dt$ to the temperature t.

If the quantity of heat instead of being a unit had been e, its motive power produced would have had for its value

$$edu = eF'tdt. \quad . \quad . \quad . \quad . \quad (4)$$

But edu is the same thing as δr; both are the power developed by the fall of the quantity e of heat from the temperature $t + dt$ to the temperature t; consequently,

$$edu = \delta r,$$

and from equations (3), (4),

$$eF'tdt = N \log v dt;$$

or, dividing by $F'tdt$,

$$e = \frac{N}{F't} \log v = T \log v.$$

Calling T the fraction $\frac{N}{F't}$ which is a function of t only, the equation

$$e = T \log v$$

is the analytical expression of the law stated pp. 80, 81. It is common to all gases, since the laws of which we have made use are common to all.

If we call s the quantity of heat necessary to change the air that we have employed from the volume 1 and from the temperature zero to the volume v and to the temperature t, the difference between s and e will be the quantity of heat required to bring the air at the volume 1 from zero to t. This quantity depends on t alone; we will call it U. It will be any function whatever of t. We shall have

$$s = e + U = T \log v + U.$$

If we differentiate this equation with relation to t alone, and if we represent it by T' and U', the differential coefficients of T and U, we shall get

$$\frac{ds}{dt} = T' \log v + U'; \quad . \quad . \quad . \quad (5)$$

$\frac{ds}{dt}$ is simply the specific heat of the gas under constant volume, and our equation (1) is the analytical expression of the law stated on page 86.

If we suppose the specific heat constant at all temperatures (hypothesis discussed above, page 92), the quantity $\frac{ds}{dt}$ will be independent of t; and in order to satisfy equation (5) for two particular values of v, it will be necessary that T' and U' be independent of t; we shall then have $T' = C$, a constant quantity. Multiplying T' and C by dt, and taking the integral of both, we find

$$T = Ct + C_1;$$

but as $T = \frac{N}{F't}$, we have

$$F't = \frac{N}{T} = \frac{N}{Ct + C_1}.$$

Multiplying both by dt and integrating, we have

$$Ft = \frac{N}{C} \log (Ct + C_1) + C_2;$$

or changing arbitrary constants, and remarking further that Ft is 0 when $t = 0°$,

$$Ft = A \log \left(1 + \frac{t}{B}\right) \quad . \quad . \quad . \quad . \quad (6)$$

The nature of the function Ft would be thus

determined, and we would thus be able to estimate the motive power developed by any fall of heat. But this latter conclusion is founded on the hypothesis of the constancy of the specific heat of a gas which does not change in volume—an hypothesis which has not yet been sufficiently verified by experiment. Until there is fresh proof, our equation (6) can be admitted only throughout a limited portion of the thermometric scale.

In equation (5), the first member represents, as we have remarked, the specific heat of the air occupying the volume v. Experiment having shown that this heat varies little in spite of the quite considerable changes of volume, it is necessary that the coefficient T' of $\log v$ should be a very small quantity. If we consider it nothing, and, after having multiplied by dt the equation

$$T' = 0,$$

we take the integral of it, we find

$$T = C, \quad \text{constant quantity;}$$

but

$$T = \frac{N}{F't},$$

whence

$$F't = \frac{N}{T} = \frac{N}{C} = A;$$

whence we deduce finally, by a second integration,

$$Ft = At + B.$$

As $Ft = 0$ when $t = 0$, B is 0; thus

$$Ft = At;$$

that is, the motive power produced would be found to be exactly proportional to the fall of the caloric. This is the analytical translation of what was stated on page 98.

NOTE F.—M. Dalton believed that he had discovered that the vapors of different liquids at equal thermometric distances from the boiling-point possess equal tensions; but this law is not precisely exact; it is only approximate. It is the same with the law of the proportionality of the latent heat of vapors with their densities (see Extracts from a Mémoire of M. C. Despretz, *Annales de Chimie et de Physique*, t. xvi. p. 105, and t. xxiv. p. 323). Questions of this nature are closely connected with those of the motive power of heat. Quite recently MM. H. Davy and Faraday, after having conducted a series of elegant experiments on the liquefaction of gases by means of considerable pressure, have tried to observe the changes of tension of these liquefied gases on account of slight changes of temperature. They have in view the application of the new liquids to the production of motive power (see *Annales de Chimie et de Physique*, January, 1824, p. 80).

According to the above-mentioned theory, we can foresee that the use of these liquids would present no advantages relatively to the economy of heat. The advantages would be found only in the lower temperature at which it would be possible to work, and in the sources whence, for this reason, it would become possible to obtain caloric.

NOTE G.—This principle, the real foundation of the theory of steam-engines, was very clearly developed by M. Clement in a memoir presented to the Academy of Sciences several years ago. This Memoir has never been printed, and I owe the knowledge of it to the kindness of the author. Not only is the principle established therein, but it is applied to the different systems of steam-engines actually in use. The motive power of each of them is estimated therein by the aid of the law cited page 92, and compared with the results of experiment.

The principle in question is so little known or so poorly appreciated, that recently Mr. Perkins, a celebrated mechanician of London, constructed a machine in which steam produced under the pressure of 35 atmospheres—a pressure never before used—is subjected to very little expansion of volume, as any one with the least knowledge of this machine can understand. It consists of a single cylinder of very small dimensions, which at each

stroke is entirely filled with steam, formed under the pressure of 35 atmospheres. The steam produces no effect by the expansion of its volume, for no space is provided in which the expansion can take place. It is condensed as soon as it leaves the small cylinder. It works therefore only under a pressure of 35 atmospheres, and not, as its useful employment would require, under progressively decreasing pressures. The machine of Mr. Perkins seems not to realize the hopes which it at first awakened. It has been asserted that the economy of coal in this engine was $\frac{9}{10}$ above the best engines of Watt, and that it possessed still other advantages (see *Annales de Chimie et de Physique*, April, 1823, p. 429). These assertions have not been verified. The engine of Mr. Perkins is nevertheless a valuable invention, in that it has proved the possibility of making use of steam under much higher pressure than previously, and because, being easily modified, it may lead to very useful results.

Watt, to whom we owe almost all the great improvements in steam-engines, and who brought these engines to a state of perfection difficult even now to surpass, was also the first who employed steam under progressively decreasing pressures. In many cases he suppressed the introduction of the steam into the cylinder at a half, a

third, or a quarter of the stroke. The piston completes its stroke, therefore, under a constantly diminishing pressure. The first engines working on this principle date from 1778. Watt conceived the idea of them in 1769, and took out a patent in 1782.

We give here the Table appended to Watt's patent. He supposed the steam introduced into the cylinder during the first quarter of the stroke of the piston; then, dividing this stroke into twenty parts, he calculated the mean pressure as follows:

Portions of the descent from the top of the cylinder.		Decreasing pressure of the steam, the entire pressure being 1.	
0.05	Steam arriving freely from the boiler.	1.000	Total pressure.
0.10		1.000	
0.15		1.000	
0.20		1.000	
Quarter.....0.25		1.000	
0.30	The steam being cut off and the descent taking place only by expansion.	0.830	
0.35		0.714	
0.40		0.625	
0.45		0.555	
Half........0.50		0.500	Half original pressure.
0.55		0.454	
0.60		0.417	
0.65		0.385	
0.70		0.375	
0.75		0.333	One third.
0.80		0.312	
0.85		0.294	
0.90		0.277	
Bottom of 0.95		0.262	
cylinder...1.00		0.025	Quarter.
		Total, 11.583	

$$\text{Mean pressure } \frac{11.583}{20} = 0.579.$$

On which he remarked, that the mean pressure is more than half the original pressure; also that in employing a quantity of steam equal to a quarter, it would produce an effect more than half.

Watt here supposed that steam follows in its expansion the law of Mariotte, which should not be considered exact, because, in the first place, the elastic fluid in dilating falls in temperature, and in the second place there is nothing to prove that a part of this fluid is not condensed by its expansion. Watt should also have taken into consideration the force necessary to expel the steam which remains after condensation, and which is found in quantity as much greater as the expansion of the volume has been carried further. Dr. Robinson has supplemented the work of Watt by a simple formula to calculate the effect of the expansion of steam, but this formula is found to have the same faults that we have just noticed. It has nevertheless been useful to constructors by furnishing them approximate data practically quite satisfactory. We have considered it useful to recall these facts because they are little known, especially in France. These engines have been built after the models of the inventors, but the ideas by which the inventors were originally influenced have been but little understood. Ignorance of these ideas

has often led to grave errors. Engines originally well conceived have deteriorated in the hands of unskilful builders, who, wishing to introduce in them improvements of little value, have neglected the capital considerations which they did not know enough to appreciate.

Note H.—The advantage in substituting two cylinders for one is evident. In a single cylinder the impulsion of the piston would be extremely variable from the beginning to the end of the stroke. It would be necessary for all the parts by which the motion is transmitted to be of sufficient strength to resist the first impulsion, and perfectly fitted to avoid the abrupt movements which would greatly injure and soon destroy them. It would be especially on the working beam, on the supports, on the crank, on the connecting-rod, and on the first gear-wheels that the unequal effort would be felt, and would produce the most injurious effects. It would be necessary that the steam-cylinder should be both sufficiently strong to sustain the highest pressure, and with a large enough capacity to contain the steam after its expansion of volume, while in using two successive cylinders it is only necessary to have the first sufficiently strong and of medium capacity,—which is not at all difficult,—and to have

the second of ample dimensions, with moderate strength.

Double-cylinder engines, although founded on correct principles, often fail to secure the advantages expected from them. This is due principally to the fact that the dimensions of the different parts of these engines are difficult to adjust, and that they are rarely found to be in correct proportion. Good models for the construction of double-cylinder engines are wanting, while excellent designs exist for the construction of engines on the plan of Watt. From this arises the diversity that we see in the results of the former, and the great uniformity that we have observed in the results of the latter.

NOTE I.—Among the attempts made to develop the motive power of heat by means of atmospheric air, we should mention those of MM. Niepce, which were made in France several years ago, by means of an apparatus called by the inventors a pyréolophore. The apparatus was made thus: There was a cylinder furnished with a piston, into which the atmospheric air was introduced at ordinary density. A very combustible material, reduced to a condition of extreme tenuity, was thrown into it, remained a moment in suspension in the air, and then flame was applied. The inflammation pro-

duced very nearly the same effect as if the elastic fluid had been a mixture of air and combustible gas, of air and carburetted hydrogen gas, for example. There was a sort of explosion, and a sudden dilatation of the elastic fluid—a dilatation that was utilized by making it act upon the piston. The latter may have a motion of any amplitude whatever, and the motive power is thus realized. The air is next renewed, and the operation repeated.

This machine, very ingenious and interesting, especially on account of the novelty of its principle, fails in an essential point. The material used as a combustible (it was the dust of Lycopodium, used to produce flame in our theatres) was so expensive, that all the advantage was lost through that cause; and unfortunately it was difficult to employ a combustible of moderate price, since a very finely powdered substance was required which would burn quickly, spread rapidly, and leave little or no ash.

Instead of working as did MM. Niepce, it would seem to us preferable to compress the air by means of pumps, to make it traverse a perfectly closed furnace into which the combustible had been introduced in small portions by a mechanism easy of conception, to make it develop its action in a cylin-

der with a piston, or in any other variable space; finally, to throw it out again into the atmosphere, or even to make it pass under a steam-boiler in order to utilize the temperature remaining.

The principal difficulties that we should meet in this mode of operation would be to enclose the furnace in a sufficiently strong envelope, to keep the combustion meanwhile in the requisite state, to maintain the different parts of the apparatus at a moderate temperature, and to prevent rapid abrasion of the cylinder and of the piston. These difficulties do not appear to be insurmountable.

There have been made, it is said, recently in England, successful attempts to develop motive power through the action of heat on atmospheric air. We are entirely ignorant in what these attempts have consisted—if indeed they have really been made.

NOTE J.—The result given here was furnished by an engine whose large cylinder was 45 inches in diameter and 7 feet stroke. It is used in one of the mines of Cornwall called Wheal Abraham. This result should be considered as somewhat exceptional, for it was only temporary, continuing but a single month. Thirty millions of lbs. raised one English foot per bushel of coal of 88 lbs. is generally regarded as an excellent result for steam-engines.

It is sometimes attained by engines of the Watt type, but very rarely surpassed. This latter product amounts, in French measures, to 104,000 kilograms raised one metre per kilogram of coal consumed.

According to what is generally understood by one horse-power, in estimating the duty of steam-engines, an engine of ten horse-power should raise per second 10×75 kilograms, or 750 kilograms, to a height of one metre, or more, per hour; $750 \times 3600 = 2,700,000$ kilograms to one metre. If we suppose that each kilogram of coal raised to this height 104,000 kilograms, it will be necessary, in order to ascertain how much coal is burnt in one hour by our ten-horse-power engine, to divide 2,700,000 by 104,000, which gives $\frac{2700}{104} = 26$ kilograms. Now it is seldom that a ten-horse-power engine consumes less than 26 kilograms of coal per hour.